

## Chapter 2

### What Is Harmonic Resonance?

Harmonic resonance is an extraordinarily diverse and varied phenomenon seen in countless forms throughout the universe, from gravitational orbital resonances, to electromagnetic oscillations, to acoustical vibrations in solids, liquids, and gases, to laser resonance in light and microwaves. Harmonic resonance spans a vast range of spatial scales, from the tiniest wave-like vibrations of the elemental particles of matter, to orbital resonances that emerge from spinning disks of gas and stars. But across this vast range of spatial scales and diverse media, there are certain general properties of harmonic resonance that are common to all of them. They all tend to oscillate at some characteristic frequency, and at its higher harmonics, frequencies that are integer multiples of the fundamental frequency. They all exhibit spatial standing waves, whose wavelength is inversely proportional to their frequencies. They all tend to subdivide one, two, or three-dimensional spaces into equal intervals of alternating reciprocating forces dynamically balanced against each other, with the twin properties of periodicity and symmetry across every possible dimension of space and time. These, and many other properties, are properties of resonance in the abstract, manifested across all those diverse forms and media. Harmonic resonance is a higher order organizational principle of physical matter, that transcends any particular implementation in a physical medium. It is the properties of that transcendent, more general concept of harmonic resonance that are the focus of this book, because it is those transcendent properties that reveal the essential properties of resonance itself, and explain how those properties lead to the emergence of mind from brain.

The minimal prerequisite for harmonic resonance is some system that when deflected from some rest state, or equilibrium condition, experiences a restoring force that pushes it back toward that equilibrium state. Also required is some kind of inertia, or momentum term, that makes it overshoot the equilibrium point and pass on through, continuing on to a deflection of equal magnitude in the opposite direction, from which point the restoring force will accelerate the system back toward the equilibrium center again, setting up for repeating back and forth oscillations that can continue indefinitely in the absence of frictional losses.

The simplest harmonic resonances can be found in highly constrained dynamic systems, like a pendulum that is free to swing only within a plane, or a linear mass-and-spring system sliding back and forth on a frictionless surface. This kind of resonance is known as *simple harmonic motion*, and it has a number of beautifully harmonious aspects or symmetries. The position-time trace of a swinging pendulum or mass-and-spring system describes a sinusoid back and forth across an equilibrium point, with a constant and continuous reciprocal exchange between potential and kinetic energy. The sinusoid is circular motion in projection, constantly accelerating up and down at a rate that itself follows a sinusoidal function, an acceleration profile that is 90 degrees phase-advanced to the motion it induces. It is a perfectly regular curve that follows a simple law of acceleration with a harmonious dynamic geometry.

The equation for simple harmonic motion is given by

$$x(t) = A \sin(2\pi ft + \delta) \quad (\text{EQ 1})$$

where  $x(t)$  is the displacement from the origin at time  $t$ ,  $A$  is the amplitude of oscillation,  $f$  is the frequency, and  $\delta$  is the phase of the oscillation. Differentiating once gives an expression for the velocity at any time.

$$v(t) = \frac{d}{dt}x(t) = A\omega \cos(\omega t + \delta) \quad (\text{EQ 2})$$

and differentiating again gives the acceleration at any time.

$$a(t) = \frac{d^2}{dt^2}x(t) = -A\omega^2 \sin(\omega t + \delta) \quad (\text{EQ 3})$$

The mathematical characterization of simple harmonic motion as a sinusoidal oscillation that repeats exactly in each cycle, captures the constant unchanging aspect of harmonic resonance. But some of the most interesting aspects of resonance occur as a distortion of that endless pattern, as the resonance resists the distortion and tries to restore the symmetry of the perfect periodic pattern. In the phenomenon of entrainment, two oscillators, like pendulum clocks hung next to each other on a wall, will subtly distort each other's oscillations a bit at a time, bending and warping each sinusoidal time trace until they are swinging in lock-step counterphase harmony.

The simple harmonic oscillator has another peculiarity that is of significance: it responds not only to periodic forces applied at its natural fundamental frequency, but it also responds to higher harmonics of that frequency. For example if a pendulum, initially motionless, is tapped periodically at exactly double its fundamental frequency at the moment it reaches the equilibrium point, it will begin to swing half-cycles, making repeated excursions in one direction only, to reverse abruptly at the equilibrium point as it rebounds off the next tap. Although it takes precise tapping at precisely the right time and strength to achieve this kind of motion, this peculiar property of the simple harmonic oscillator opens the possibility for setting up pairs of identical pendulums swinging against each other, colliding and rebounding off each other across the equilibrium point, creating a double oscillation of mirror-symmetric motions at double the fundamental frequency. In fact, any number of simple harmonic oscillators can be strung together in this manner to create compound oscillator systems. For example mass-and-spring oscillators can be chained together into a string of masses connected by springs, each one a simple harmonic oscillator, but together they form a complex compound oscillator that exhibits many more levels of resonance than the simple harmonic components of which it is composed.

### **Lissajous Figures**

Simple harmonic motion gets a lot more interesting when you allow it a second dimension of freedom. For example a pendulum that is free to swing in both x and y dimensions can describe all kinds of complex elliptical orbits that continuously exchange potential and kinetic energy across both x and y dimensions. A similar two-dimensional oscillation is seen in the *Lissajous curves* on an oscilloscope, achieved by plotting two sinusoidal oscillations against each other, one in x and the other in y, as shown in Figure 2.1. The first row in Figure 2.1 shows two sinusoidal oscillations of the same frequency plotted against each other, with a range of phase shifts between the two oscillations along the columns from 0 to  $\pi$  in increments of  $\pi/8$ . At zero phase difference the plot oscillates back and forth along the  $y = x$  diagonal line. At a phase shift of  $\pi/2$  (or 90 degrees) the plot oscillates round and round a perfect circle, and at a phase shift of  $\pi$  (180 degrees) the plot forms a diagonal line tilted the other way, along the  $y = (-x)$  line. Subsequent rows in Figure 2.1 plot two sinusoids of different frequencies, where the frequency of the x oscillation is either an integer multiple (2, 3, 4) or a rational fraction ( $1/2$ ,  $3/2$ ,  $4/3$ ) of the frequency of y.

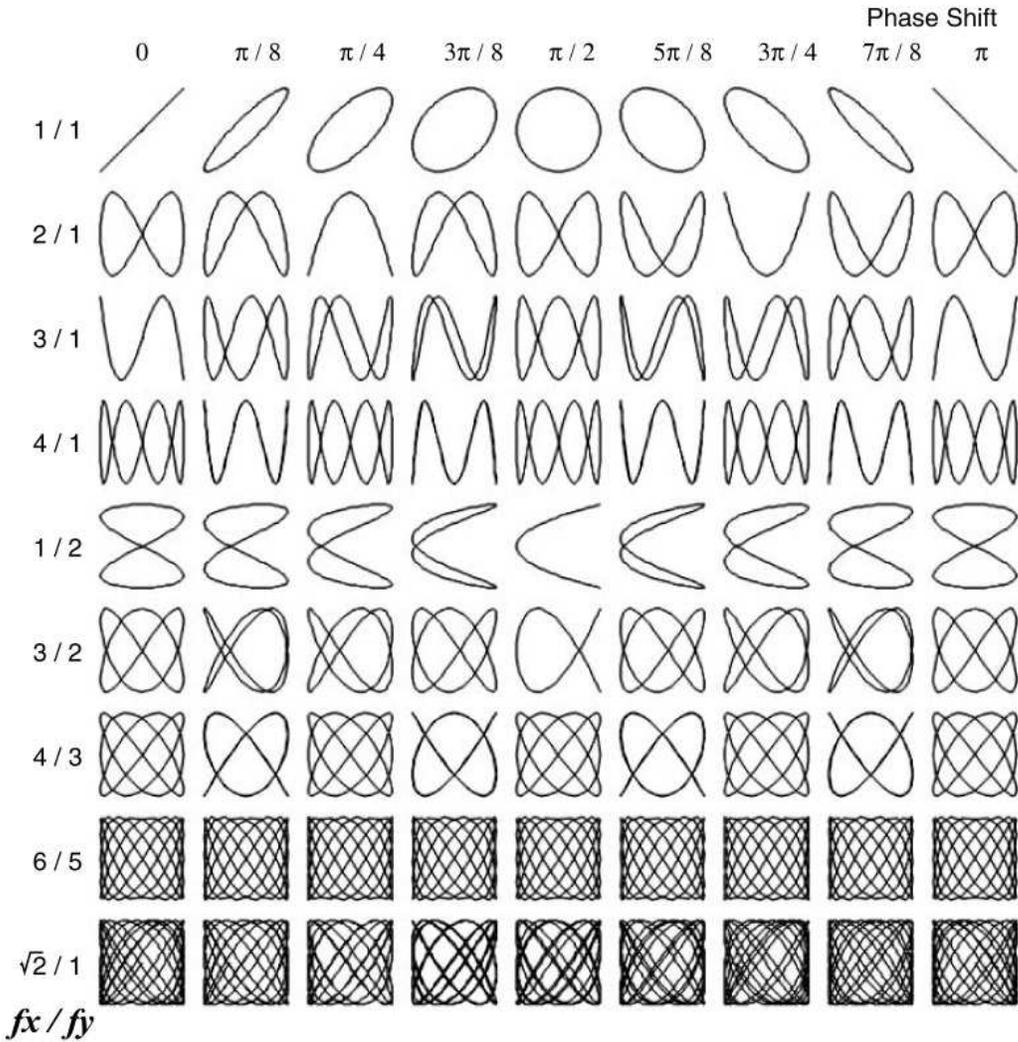


Fig. 2.1. Lissajous figures, created by plotting one sinusoid in  $x$  against another in  $y$ . The columns show various phase shifts between the two oscillations, in increments of  $\pi/8$ . The rows show the effect of varying the frequency of  $x$  relative to that of  $y$  in integer ratios, to create closed figures. The last row shows an irrational ratio that defines an open figure that covers new ground for ever until the whole plot turns black.

If the frequencies of the  $x$  and  $y$  oscillations are matched, with a 90 degree phase lag between them, the Lissajous figure forms a circle. If the frequency of one is exactly double that of the other, it produces a figure 8 shape. Any other integer relation between the two frequencies produces other closed lissajous curves, like those in Figure. However only harmonically related frequencies form closed trajectories, or static Lissajous figures. If the frequency  $f_x$  is not an integer multiple or fraction of  $f_y$  then the pattern will cycle endlessly across the scope, a travelling wave rather than a standing wave, never quite retracing exactly the same pattern. This is shown for a partial trace in the last row of Figure 2.1, that plots a frequency

ratio of the irrational fraction square root of two. The plot *almost* retraces its path each cycle of the oscillation, but not quite, and if allowed to run forever, the plot would eventually fill the figure entirely with a solid black field when the trace lines get close enough to abut each other, although mathematically it would always be covering new ground at ever finer scale. Once again we see a very simple system composed of two independent oscillators, that produces a fantastically complex repertoire of beautiful periodic and/or symmetrical patterns when combined.

Lissajous figures can also be produced in three dimensions by plotting sinusoidal oscillations in  $x$ ,  $y$ , and  $z$  dimensions, some of which are shown in Figure 2.2, where they are displayed in a grid sorted by their frequencies in the  $x$ ,  $y$ , and  $z$  dimensions.. Again, only integer ratio frequencies are shown, because they are

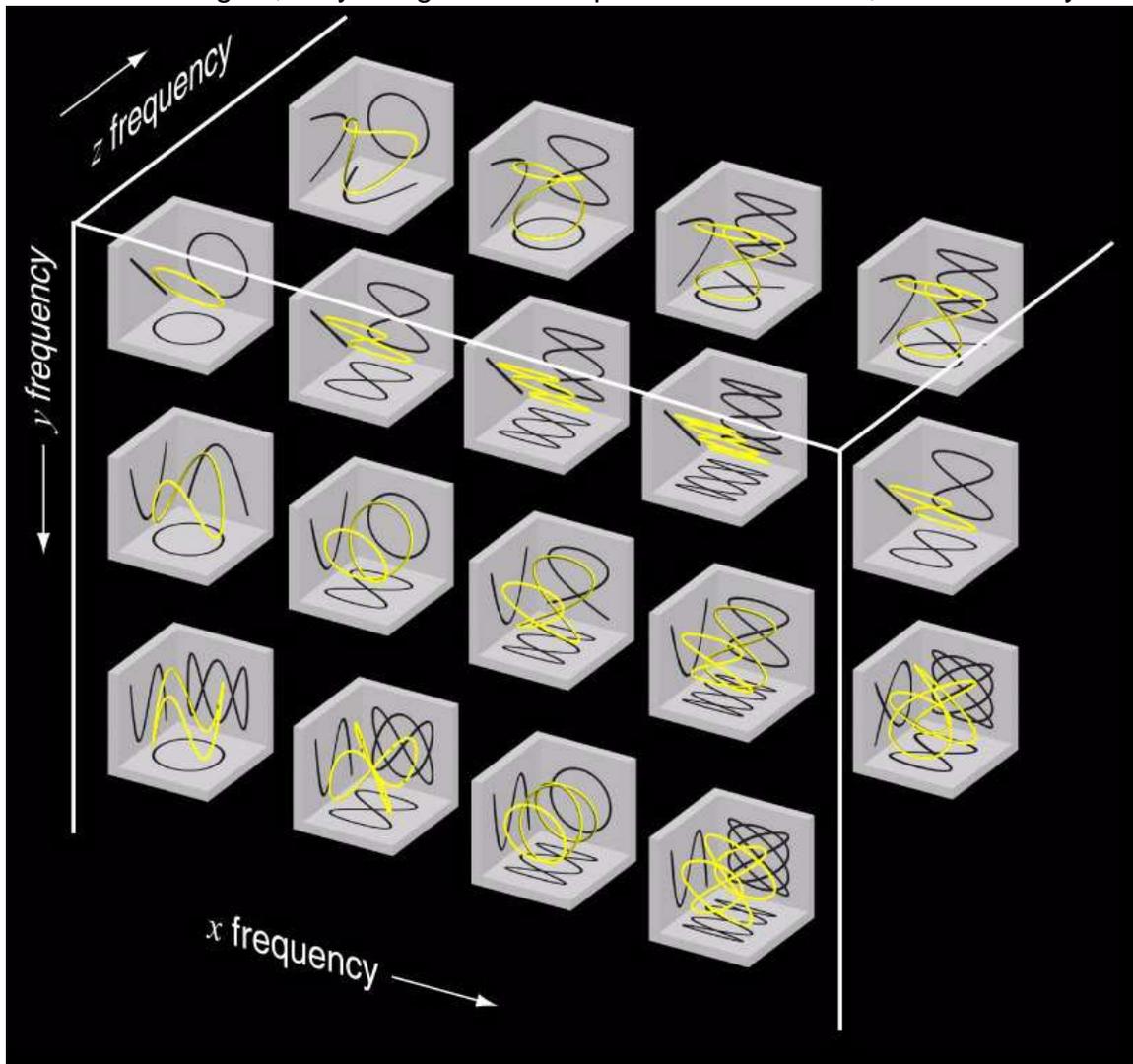


Figure 2.2. Lissajous figures in 3-D, displayed in three-dimensional grid arranged by  $x$ ,  $y$ ,  $z$  frequencies in the ranges  $x \{1-4\}$ ,  $y \{1-4\}$  and  $z \{1-2\}$ . The phase of  $y$  is arbitrarily advanced by  $\pi/2$  relative to  $x$ , while the phase of  $z$  is retarded by  $\pi/2$ . Other phase ratios produce even more arrays of closed Lissajous figures than those shown here.

the only ones that produce closed figures. Also, in this figure only one relative phase relation is shown between the  $x$ ,  $y$ , and  $z$  waveforms, (arbitrarily, the phases of  $y$  and  $z$  are advanced and retarded by  $\pi/2$  relative to that of  $x$ , respectively). Other phase ratios (in rational fractions of  $2\pi$ , as in Figure 2.1) produce still more arrays of closed patterns beyond those shown in this figure. Again, we have a very simple system of three oscillations that together define an enormously complex array of periodic and symmetrical patterns in a lawfully organized hierarchy. There are always many more irregular, or open figures in between the symmetrical closed figures shown in Figure 2.1.

Physically, these same kinds of patterns can be achieved quite simply by suspending a mass on springs, as suggested in Figure 2.3 A, and twanging it like

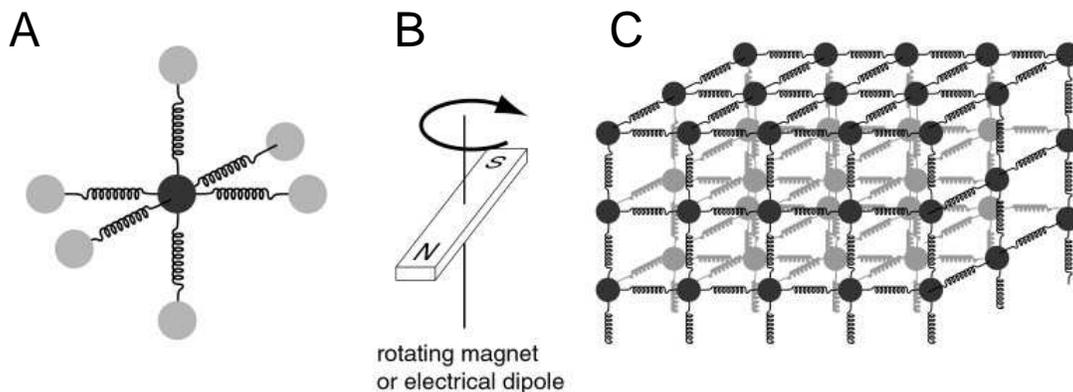


Fig. 2.3. A: A mass suspended on springs will naturally tend to oscillate in three-dimensional Lissajous figures. B: If a bar magnet, or electrostatic dipole, is rotated in the vicinity of a sprung mass that is magnetized, or electrically charged, respectively, it will set the mass into vibration at the right rotational frequency. C: A block of sprung masses of that sort will also respond to a rotating magnet or dipole either in random, thermal vibration, or in coherent oscillations, by the same principle that electromagnetic radiation is absorbed by solids, when its frequency matches the natural modes of oscillation of the system.

a guitar string to set it into vibration. This creates a compound oscillator whose natural modes of vibration correspond to the family of three-dimensional Lissajous figures. Actually, a single mass-and-springs system will naturally oscillate not only at the discrete integer ratio frequencies of the closed Lissajous figures, but also at any of the many intermediate irregular frequency ratios of the travelling wave Lissajous figures, although more complex compound systems can possess emergent interactions that preferentially promote the harmonic oscillations of the Lissajous figures, as explained below.

## Molecular Vibrations

Atoms that are locked in solids each act as simple harmonic oscillators in three dimensions, much like a mass held in place by springs, as depicted in Figure 3 A, as each atom in the glass or crystal lattice is held in its place by elastic forces of attraction and repulsion. If an atom in a solid is “twanged”, or knocked out of its central equilibrium position, it will automatically follow an oscillatory path that traces out a three-dimensional Lissajous figure, like a mass-and-springs system.

The electrostatic forces that hold atoms in their molecular places, act over a quite limited range. But atoms also interact across much longer distances by the principle of entrainment and resonance. An agitated atom in a lattice vibrates energetically at its natural resonant frequency, and that tiny oscillating electrostatic charge radiates energy outward in all directions as an electromagnetic field, that appears in the form of a miniscule, but constantly reciprocating force of modulated attraction or repulsion. That tiny alternating force can be detected by a distant atom that happens to share exactly the same resonant frequency, because the tiny pushes of the alternating field accumulate over time, as when pushing a child on a swing with carefully timed shoves, and thus the effect of that reciprocating field is magnified by orders of magnitude in influence, compared to the static force of attraction or repulsion, whose effect is negligible except at very short range. This principle can be demonstrated by analogy, by making the sprung mass of Figure 2.3 A out of a magnet, or charged mass, and setting up a bar magnet, or electrostatic dipole, in the vicinity of that sprung mass, mounted on a pivot so that it can be rotated about its axis, as suggested in Figure 2.3 B. Although the position of the sprung mass is barely influenced by the rotation of the bar magnet to different orientations, if the bar magnet is rotated continuously at exactly the resonant frequency of the sprung mass, it will set the sprung mass into vibration. This is the principle behind the emission and absorption of electromagnetic radiation by atoms and molecules of matter, which is itself a resonance effect. The different properties or effects of the various bands of electromagnetic radiation on physical matter, such as gamma rays, X-rays, ultra-violet, visible light, infra-red, microwaves, and radio waves, is due to the fact that different frequencies resonate with different components or groupings of material substance. Gamma rays and x-rays are the highest frequency radiation, with enough photon energy to strip the electrons off atoms altogether, ionizing the absorbing matter into positive nuclei and free electrons, with destructive consequences to fragile molecular structures like biological tissue. Ultra-violet radiation is absorbed by the lowest level electron shells,

nearest to the atomic nucleus, which also tends to ionize the atoms and thus knock atoms out of their places, whereas visible light is absorbed chiefly by the higher level electrons, raising them to higher energy levels, from whence they spontaneously decay again, releasing another photon by re-emission, without disrupting the underlying molecular structures. Infra-red radiation is absorbed by whole molecules, causing them to oscillate as thermal vibrations, or heat. Microwave radiation causes molecules to rotate or twist in place in their lattice locations. Radio waves are so low in frequency that they generally pass through bulk matter without interacting with it, which is why you can listen to your radio inside your house, since the waves pass transparently through the walls of your house. But radio waves are absorbed by metals, like the antenna of your radio, and the reason for this is also an interesting resonance effect.

The electrons in a non-metal are tightly bound to their host atoms, and thereby locked into the glass or crystal lattice. This is also true for most of the electrons of a metal. But metals, by their nature, have extra electrons in their partially-filled outermost electron shells that are not so tightly bound to their host atoms, but can wander about freely through the bulk material of the metal. These free electrons flow between the atoms in the lattice as a continuous “sea” of electrons, that behaves much like a fluid, although that negatively charged electrical fluid remains trapped strictly within the confines of the bulk of the metal solid, because if an electron were to escape, the bulk matter would instantly become positively charged, and that positive charge would instantly suck the errant electron (or another one like it) back into the metal solid to restore the electrical balance. This “sea” of electrons thus behaves like an emergent larger object, an elastic object on the size scale of the bulk material itself. The resonant frequency of this “sea” of electrons, therefore, is not a property of the individual electrons of which it is composed, but rather, it is a property of the larger global entity into which they are seamlessly merged. The entire antenna, in effect, behaves like a single electrical entity, with a resonant frequency that is a function of the size and shape of the whole antenna, rather than of its component atoms or electrons. That is why antennas have to be carefully tuned to match the frequency of the radiation that they are designed to receive, or to transmit, with longer antennas used for long wave radio, and progressively shorter antennas for short wave, VHF (very high frequency), and UHF (ultra high frequency) radio waves.

The principle of resonances of bulk matter can also be demonstrated by analogy, by setting up an array of masses connected by springs, as suggested in Figure

2.3 C. Although each mass has its own resonant frequency, as in Figure 2.3 A, the masses are not independent, but intercoupled by the array of springs. If a springy array of masses of this sort is “twanged”, it will tend to wobble like a jelly, as the vibration is communicated from atom to atom throughout the bulk matter. The oscillatory behavior of such a system depends not so much on the forces on its individual atoms, but more on the bulk properties of the array as a whole, in particular, its size and shape. This is emergence of yet another form, the resonant properties of the whole being far more than a simple sum of its component parts. The selective amplification of harmonic oscillations over chaotic or inharmonic ones stems from the fact that the chaotic, random oscillations tend to cancel each other out, whereas any coherence in the oscillations of groups of atoms will feed back on itself each cycle, which eventually produces global coherent oscillations from the energy of initially random noise. Crystals of solid matter also vibrate in this holistic manner, with coherent oscillations coursing back and forth across the crystal as a whole, at a frequency that is determined by the bulk geometry of the crystal. This is the principle behind the crystal oscillators that are used in electronic watches, and to time the data cycles in digital computers.

A crystal oscillator is a tiny piece of crystal, often quartz, mounted between electrical plates. The crystal is set into electro-mechanical oscillation by a randomly alternating electrical voltage across the plates, by the piezoelectric effect, whereby an electrical voltage across the crystal causes a physical deformation of the crystal, and conversely, the physical deformation of the crystal causes an electrical voltage. This is the principle by which sparks are generated when knocking two pieces of quartz together. If a random-noise alternating voltage is applied across the crystal, it will set the crystal into electromechanical vibration at its resonant frequency, which in turn generates a periodic electrical voltage, similar in principle to the acoustical tone produced when blowing a stream of air across the mouth of a bottle, and to the note produced by a violin string by the random rasping of a rosined bow, and the tone produced in a bugle by blowing a rude “raspberry” into the mouthpiece, and the beautiful harmonic patterns that appear on a Chladni plate when pressing a piece of dry ice against it. In fact, the resonances of a crystal oscillator of this sort are much like the modes of a *harmonic resonance continuum*.

### **Harmonic Resonance Continuum: The Wave Equation**

The most spectacular emergence of harmonic resonance is observed when provided with the extra dimension of variability of a *spatial continuum* of vibrating

substance. For example if one end of a rope is shaken vigorously with a periodic oscillation, travelling waves propagate away from the point of shaking, each point on the rope oscillating back and forth across the average location of the rope, or equilibrium line, like a *continuous array* of simple harmonic oscillators. Adjacent points on the rope are flexibly connected so as to impose a smoothness, or continuity constraint along the rope as a whole, forming alternating waves of deflection in the rope that can run continuously like travelling waves, or can reflect off an attachment point at the far end of the rope, and thus produce standing waves. Similar waves can be produced in a slinky, by waving it up and down, which produces transverse waves, as in the rope. But the longitudinal elasticity of the slinky also allows for longitudinal waves of compression and rarefaction, stimulated by pumping the slinky back and forth in a direction parallel to its length. This is directly analogous to the pulses of compressed and rarefied air that constitute sound waves. As with the rope, the slinky can produce either travelling waves or standing waves, depending on whether the far end of the slinky is free, allowing the travelling waves to just run right off the end, or fixed to a rigid support that reflects the waves back in the opposite direction.

The propagation of waves through a continuous elastic inertial medium of this sort is modeled mathematically by the *wave equation*, developed by D'Alembert, and refined by Euler. This equation can be derived by considering the physics of a chain of masses connected by springs, using Newton's laws of motion and Hooke's law for linear springs. The back-and-forth alternating motion of each mass reveals a cyclic and reciprocating pattern of acceleration. The forces that produce this acceleration are described by Newton's second law, that is

$$F_N = ma(t) = m \frac{\partial^2}{\partial t^2} u(x, t) \quad (\text{EQ 4})$$

where  $F_N$  is the horizontal force due to Newton's law,  $m$  is the mass, and  $a(t)$  is the acceleration at time  $t$ , which can be expressed as a function of the second partial derivative of displacement  $u$  with respect to  $x$ , where  $u$  is expressed relative to its equilibrium location. The force that is causing this acceleration is the force due to the tension in the springs that connect the masses, which varies with the distance between adjacent masses by Hooke's Law, that is,

$$F_H = F_{x-h} + F_{x+h} = k[u(x-h, t) - u(x, t)] + K[u(x+h, t) - u(x, t)] \quad (\text{EQ 5})$$

where  $F_H$  is the total spring force on the mass, which is the sum of the spring forces from the two adjacent neighbors, which in turn is equal to spring constant  $k$  times the difference in displacement between the central mass and its two adjacent neighbors. This allows the following equation of force as mass times acceleration.

$$m \frac{\partial^2}{\partial t^2} u(x, t) = k[u(x-h, t) - u(x, t) + u(x+h, t) - u(x, t)] \quad (\text{EQ 6})$$

If the array of masses consists of  $N$  masses spaced evenly over the length  $L = Nh$  of total mass  $M = Nm$ , and the total stiffness of the array  $K = k/N$  we can rewrite the above equation as:

$$\frac{\partial^2}{\partial t^2} u(x, t) = \frac{KL^2}{M} \cdot \frac{[u(x-h, t) - 2u(x, t) + u(x+h, t)]}{h^2} \quad (\text{EQ 7})$$

We can now convert this problem from the discrete case, with  $N$  distinct masses separated by a distance  $h$  between masses, to a continuous case more like the equation for a slinky, where the mass and the spring force are distributed uniformly across the length of the slinky spring, that is, in the limit as  $N \rightarrow \infty$ , and  $h \rightarrow 0$ , we have

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad (\text{EQ 8})$$

where  $u$  is the degree of deviation from the equilibrium location,  $x$  is the distance along the linear medium, and  $c$  is the speed of wave propagation through the medium. If the displacement  $u$  is defined as a vertical displacement, at right angles to the direction of wave propagation, the wave equation applies to transverse waves, whereas when  $u$  is defined as a longitudinal displacement parallel to the direction of wave propagation, the same equation applies to longitudinal waves. In fact, the wave equation applies to a vast range of resonating systems, from mechanical vibrations, to electromagnetic oscillations of light, radio waves, microwaves, to sound waves, and a slightly modified form of the wave equation is seen in the Schrödinger equation that defines the quantum resonances of atoms, electrons, and sub-atomic particles. The wave equation captures the very essence of the higher order organizational principle behind

harmonic resonance that is common to all of the diverse physical manifestations of resonance.

Let us examine the meaning of the wave equation, using the example of air pressure waves in a linear tube, like those in a flute when a note is blown. Figure 2.4 A and B depict the instantaneous air pressure in a tube, open at both ends, at the two extremes of the cycle of a first harmonic oscillation. In Figure 2.4 A, there is an over-pressure at the center of the tube, indicated by the darker shading, and an under-pressure at the two ends, while in Figure 2.4 B there is an under-pressure at the center, and over-pressure at the two ends. Figure 2.4 C and D show a plot of  $u(x)$  as a pressure or density function along the length of the tube. Figure 2.4 E and F show the partial derivative of this pressure function with respect to  $x$ , which is the *gradient* of the air pressure, positive where the gradient of  $u(x)$  is rising with distance  $x$  along the tube, and negative where it is falling. Note how the gradient  $du/dx$  is zero at both the peaks and the troughs of the air pressure function  $u(x)$ . Figure 2.4 G and H show the double partial derivative  $d^2u/dx^2$ , that is, the *gradient of the gradient* of air pressure along the tube, or how steeply the gradient is rising or falling at each point along the tube. .

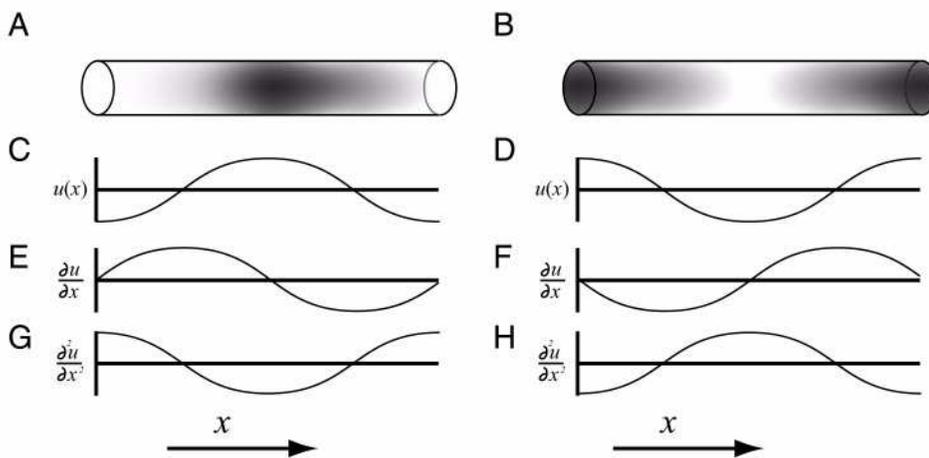


Fig. 2.4. A and B: Instantaneous air pressure function during the two extreme points in the cycle of a first harmonic standing wave in a tube, separated by 180 degrees in phase. C and D: Air pressure function expressed as  $u(x)$ . E and F: The partial spatial derivative of the pressure function. G and H: The double partial derivative of the pressure function. This is the term that equates to the double temporal derivative of pressure in the wave equation.

Now the meaning of the wave equation can be stated in words as follows: The *rate of growth of the rate of growth*, or *accelerative increase* of air pressure at any point in the tube over time, ( $\delta^2u/\delta t^2$ ), is proportional to the *gradient of the gradient* of air pressure with distance along the tube ( $\delta^2u/\delta x^2$ ). In Figure 2.4 A the air

pressure is high at the center of the tube, but the double derivative of air pressure is high at the ends of the tube. This means that the air pressure will be increasing acceleratively over time at the ends of the tube, and decreasing at the center, whereas in Figure 2.4 B where the pressure is high at the ends, the pressure will be increasing acceleratively over time at the center, and falling at the ends. This equation reveals harmonic resonance to be a *balance of accelerations in space against accelerations in time*. This deserves a little explanation. In a static air system air pressure  $u(x)$  tends to distribute itself uniformly through the tube producing equal pressure throughout, as when a flute is lying quietly on a table. In a dynamic system with constant flow, (no acceleration) a gradient of air pressure,  $du/dx$ , will tend to establish itself along the length of the tube, for example when air is flowing along the tube due to a pressure difference between the two ends, as when blowing into a flute without making a sound. But the variations in air pressure due to harmonic resonance are a function of the *acceleration* of the air in the tube,  $(\delta^2 u / \delta t^2)$ . The transient and alternating patterns of high and low air pressure in the tube are sustained by synchronously oscillating symmetrically opposed patterns of accelerations of air along the tube. For example the transient and momentary peak of positive pressure depicted in Figure 2.5 A, is caused by a decelerating collision between moving slugs of air arriving at that point from opposite directions, followed by an immediate rebound, shown in Figure 2.5 B, where those slugs of air recoil in opposite directions, creating a transient and momentary trough of negative pressure at that point and peaks at the ends. This in turn sucks the air back in toward the center, setting up for repeating cycles of alternating high and low pressure disposed as a spatiotemporal pattern of standing waves, periodic in space and time. The spatial continuity and uniform momentum and elasticity of the vibrating medium distribute the patterns of acceleration across continuous sinusoidally varying patterns of acceleration of the medium. This is the origin of the balance and symmetry properties of harmonic resonance. If the colliding and rebounding slugs of air were not perfectly and symmetrically balanced, for example if the slug on one side were slightly more massive than the one on the other side, then that would instantly and immediately relocate the collision point to occur where each slug has completely exhausted its momentum against the other, where the slugs of air are again balanced. This is the essential pattern formation principle behind standing waves of harmonic resonance, that is responsible for the regularity or order of the emergent pattern.

The laws of physics, in particular, Newton's second law of motion, and Hooke's law of elastic springs, dictate that the waveforms depicted in Figure 5 conform to

the wave equation, that is, those functions are a “solution” to the wave equation, because the equation remains balanced or “equal” throughout the oscillations. There are other solutions to the wave equations besides the first harmonic. A second harmonic oscillation, with two peaks of over-pressure, alternating with two troughs of under-pressure, is another solution to the wave equation. The gradients in space and time are double those of the first harmonic, resulting in double the acceleration due to those pressure gradients, and thus also double the oscillation frequency compared to the fundamental. Other solutions are found in the third, fourth, and higher harmonics at correspondingly higher frequencies of oscillation.

Even this does not exhaust the repertoire of a harmonic resonance system, because it is not only the higher harmonics which provide solutions to the wave equation, but also combinations of harmonics, or compound waveforms that are a sum of a number of harmonic components, which also provide solutions to the wave equation. Given the fact that Fourier theory shows that virtually any waveform can be expressed to arbitrary precision as a sum of component sinusoids, we see that virtually any waveform can be expressed in the form of standing waves in a harmonic resonance representation, and those patterns that have the properties of symmetry and periodicity can be most easily expressed using only the lower harmonics that require the least energy. This is the reason why symmetry and periodicity form the foundations of aesthetic patterns in visual ornament, as well as in the patterns of melody and harmony and rhythm in music. Harmonic resonance is a *multipotential* pattern formation principle, that is, the same mechanism that generates the fundamental resonance can also create a range of patterns defined by higher harmonics and combinations of higher harmonics by the same essential principle.

### **Wave Equation in Two Dimensions**

The wave equation extends naturally into two-dimensions, where it appears in the form

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u \quad (\text{EQ 9})$$

where  $\nabla^2$  is the *Laplacian*, which is defined as the sum of the unmixed second partial derivatives across  $x$  and  $y$ , that is

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad (\text{EQ 10})$$

Waves on the surface of water provide a two-dimensional spatial continuum of oscillating substance (water) with mass and inertia at every point in the surface, and a gravity-buoyancy restoring force directed towards an equilibrium level, as the water seeks to find a single level throughout the vessel.

The meaning of the wave equation in two dimensions can be expressed in the case of a water surface as follows: The rate at which the level of the water is accelerating upwards (or downwards) at any point in the surface is proportional to the gradient of the gradient of the water surface through that point, summed through  $x$  and  $y$  dimensions. To get an intuitive feel for the meaning of this equation we can go back to Figure 2.4 again, but this time consider the plot of  $u(x)$  in Figure 2.4 C and D as a plot of the vertical displacement of the water surface in the  $x$  dimension, or a picture of the actual water surface waves (ignoring the  $y$  dimension). The center of the plot in Figure 2.4 C shows the peak of a water wave, while the plot of Figure 2.4 D is centered on a wave trough. In both cases, the gradient at the center  $du/dx$  is zero, as seen in Figure 2.5 E and F. But the gradient of the gradient  $d^2u/dx^2$  of Figure 2.4 C, shown in Figure 2.4 G, is negative, because the wave is bowed upward, surrounded by lower water, and thus the vertical acceleration at that point is negative, i.e. the water level  $u(x)$  is about to start dropping, while the trough of the wave in Figure 2.4 B is dished downward, surrounded by higher water, and thus the vertical acceleration at that point is positive, it is about to start rising, as shown in Figure 2.4 H. This gradient of the gradient is calculated independently in orthogonal  $x$  and  $y$  dimensions at each point in the surface, and the resultant vertical acceleration is computed as the sum of these unmixed second partial derivatives.

The Chladni plate also demonstrates the vibration of a two-dimensional spatial continuum, this time the substance being the steel plate, whose elastic flexibility serves as the restoring force. The Chladni figures demonstrate the true potential of a standing wave representation, with its hierarchically ordered families of patterns, all sorted into rows and columns of increasing vibrational energy. This is emergence in a most impressive form, the emergence of complex spatial structure from a simple unstructured homogeneous medium.

## Wave Equation in Three Dimensions

Nature offers three dimensions of freedom for harmonic resonance, thus even more spectacular examples of standing waves can be obtained in three dimensions. The wave equation extends naturally into three dimensions in the same form as it appears in two dimensions as in equation 9, except in this case the Laplacian  $\nabla^2$  is defined as the sum of the unmixed second partial derivatives across the three dimensions of  $x$ ,  $y$ , and  $z$ , that is

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad (\text{EQ 11})$$

Figure 2.6 shows a series of standing waves of acoustical resonance in a three-dimensional box or cavity. The green and red shades in this figure represent points of higher or lower instantaneous air pressure at each point in the cavity, depicted at an instant of maximal pressure excursion from the equilibrium condition. That is, the regions of high and low pressure reverse with each half cycle of the oscillation, the high pressure regions becoming low, and the low becoming high. The full cyclic oscillation of these volumetric patterns can be seen most clearly in Paul Falstad's excellent *Math and Physics Applets* <http://www.falstad.com/>, in particular, the *Box Modes Applet* <http://www.falstad.com/modebox/>. Figure 6 was composed of static frames from Falstad's Box Modes applet. The reader is encouraged to check out this simulation on-line, and to play around with the applet to experience its full dynamic time-variant properties.

The standing wave patterns in Figure 2.5 are arranged in order of their harmonics in the  $x$  and  $y$  directions. The box in the upper left corner of Figure 2.4, or box  $[x,y] = [0,0]$ , represents the zero<sup>th</sup> harmonic, like the DC term in a Fourier representation. The rest of the top row in Figure 2.5, from  $[1,0]$  through  $[3,0]$ , represents various harmonics of oscillation in the  $x$  dimension. The first harmonic  $[1,0]$ , has a high/low pressure profile, (alternating over time with a low/high profile) in the  $x$  dimension, and it oscillates at a fundamental vibration frequency that depends on the speed of sound through the medium. The second harmonic  $[2,0]$  has a high/low/high pressure profile (alternating with low/high/low) in the  $x$  dimension, oscillating at three halves ( $3/2$ ) of the fundamental frequency; the third harmonic  $[3,0]$  has a high/low/high/low profile at twice the fundamental frequency, and so forth, to ever higher harmonics in  $x$  (not shown).

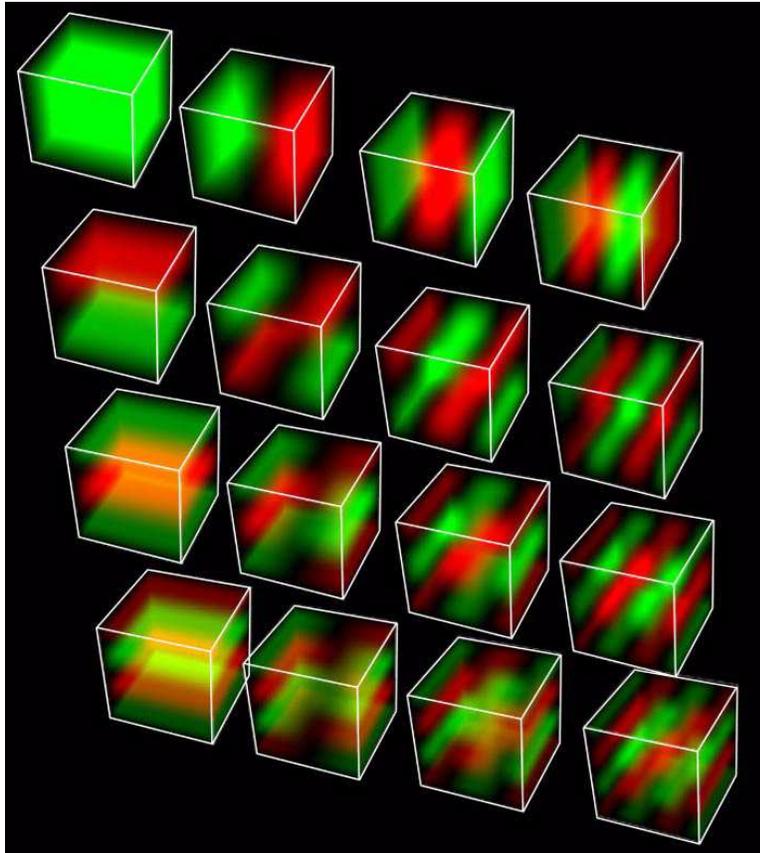


Fig. 2.5. Box modes applet from Falstad's Math and Physics Applets, showing standing waves in a cubical box, for example sound waves in an air-filled box. A rising sequence of harmonics in the  $x$  dimension is shown from column 1 through 4, and a rising sequence of harmonics in the  $y$  dimension in rows from 1 through 4. A third dimension (not shown) would complete the figure as a  $4 \times 4 \times 4$  array of the harmonics in  $x$ ,  $y$ , and  $z$ .

The left hand column in Figure 2.4, from  $[x, y] = [0,1]$  through  $[0,3]$  shows the first three harmonics of oscillation in the  $y$  dimension, from  $[0,1]$  high/low, through  $[0,3]$  high/low/high/low, this time in the vertical instead of the horizontal direction. The rest of the boxes in the array show the standing waves composed of both horizontal and vertical components. For example box  $[1,1]$  shows the first harmonic in  $x$  and  $y$  simultaneously. This defines a first harmonic of diagonal resonance in the  $[x,y]$  direction, whereas box  $[1,2]$  represents a first harmonic in  $x$ , and a second harmonic in  $y$ , and so forth. Each one of the standing waves in the array of Figure 2.5 depicts a separate and discrete pattern of standing wave oscillation that can be sustained in the box as a natural resonance.

Figure 2.5 shows only the standing waves in horizontal ( $x$ ) and vertical ( $y$ ) dimensions. Not shown are the standing waves in the depth dimension ( $z$ ) into the plane of the page. To show all the standing waves of a cubical box up to the third harmonic would require expanding the two-dimensional  $4 \times 4$  array of boxes of

Figure 2.5 into a three-dimensional  $4 \times 4 \times 4$  cubical array of boxes, with a separate and distinct standing wave pattern for every combination and permutation of  $[x, y, z]$  harmonics, an extraordinary repertoire of organized spatial patterns from a simple homogeneous resonating box. And all these geometrical organized templates, or patterns, can be called up either singly or in combinations, to produce a vast array of possible patterns, just by playing the right tone, or combinations of tones in a musical chord, in the presence of the box.

For a more concrete intuitive understanding of the principles behind these standing wave patterns, I will describe a simple physical system that could be used to produce volumetric acoustical standing waves as in Figure 2.5. We begin with a cubical air-filled box, or cavity, with three loudspeakers mounted on three orthogonal walls of the box, as shown in Figure 2.6. Each of these three speakers

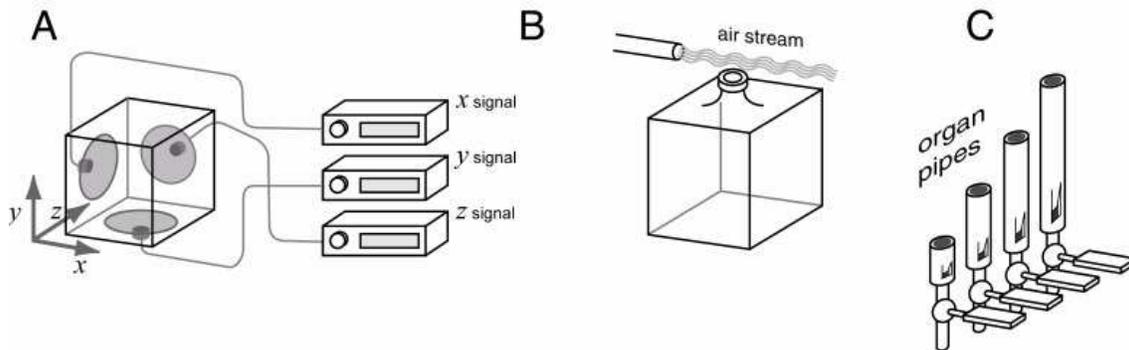


Fig. 2.6. A: A cubical air-filled box, or resonant cavity, with loudspeakers attached to three orthogonal sides, each driven by a signal generator that generates sinusoidal waves at various frequencies. B: A cubical “bottle” is made to sound by blowing air across its mouth. C: A rank of organ pipes tuned to the natural harmonics of the cubical bottle can stimulate the corresponding standing waves in the cubical resonator.

is driven by a signal generator that generates a sinusoidal waveform whose frequency and amplitude can be adjusted independently. Suppose we turn on only the signal generator for the  $x$  dimension, set at a very low frequency, and gradually sweep through a range of frequencies continuously from low to high. Wherever the frequency happens to match the fundamental vibration frequency of the box, or one of its higher harmonics, a powerful resonance will be heard in the box, as the standing wave amplifies itself by positive feedback. This will occur at discrete frequencies corresponding to the resonant modes of the box, as shown across the top row of Figure 2.5. Similar resonances can be obtained independently in the  $y$  dimension, as shown along the left hand column of Figure 2.5. The combination patterns shown in the rest of Figure 2.5 are obtained by tuning in one harmonic in the  $x$  dimension and a different harmonic in  $y$ . As in the case of Lissajous figures, these combinations will produce standing waves only

when the frequencies of the x and y dimensions are related by a rational fraction. All other ratios produce a periodic or cyclic pattern of travelling waves instead of standing waves. Even more patterns can be obtained in combination with harmonics in the z dimension, not shown in Figure 2.5.

The same standing wave patterns could also be produced in the cubical cavity much more simply by the same principle by which a note is produced in an empty bottle by blowing a stream of air across its mouth. That is, the box could be equipped with a "mouth" as suggested in Figure 2.6 B, and a stream of air can be blown across the mouth of the cubical "bottle" producing the same repertoire of standing waves as those shown in Figure 2.5. In this case the energy for the resonance is provided by the stream of air blowing across the mouth of the bottle. Acoustical resonances can also be artificially amplified, by recording the sound waves with a microphone, amplifying the signal through an amplifier, and sending the amplified sound out to a speaker near the pick-up microphone. Amplified feedback of this sort is responsible for the harsh screeches or squeals heard when the gain of a public address system is turned up too high. But the microphone and speaker can also be carefully configured to resonate across or within a resonant cavity, amplifying the standing waves that emerge in the cavity. This principle is demonstrated when an electric guitar plays a note that is endlessly sustained due to feedback, that is, the string is vibrating to the sound of its own amplified vibration, a feedback loop that goes from the physical vibration of the string, to an electrical signal which is amplified, back to an auditory sound wave that feeds back to the vibration of the string again. The amplified resonance can now be modulated by changing the resonant properties of the string, that is, by pressing it at different frets to produce different notes. This principle can be demonstrated in a resonant acoustical cavity, like a large bottle or carboy, by placing a microphone and loudspeaker at the mouth of the bottle, sending the amplified signal picked up by the microphone back out to the loudspeaker. By setting the gain just the right, the bottle can be made to howl or wail at one of its many modes of vibration. Small variations in the placement of the microphone and/or speaker, and adjustments of the gain, can make the sound "yodel" between different frequencies, like a bugle playing different notes in the same length of tube, corresponding to different standing wave modes in the resonating cavity.

## Synchronization of Distributed Oscillators

The amplification in a harmonic resonance system need not involve only a single amplifier, but can also be performed in a distributed computational architecture using hundreds or thousands of individual local amplifiers distributed uniformly throughout the resonating cavity, that instantaneously amplify and play back the acoustical signal picked up in that local region. This would also tend to amplify the natural standing wave resonances of that resonator while allowing multiple resonances to occur simultaneously. The spontaneous emergence of electrochemical resonance in bulk neural tissue could be explained by this kind of emergent process whereby each cell of the tissue behaves as a local resonator that is capable of resonating at a range of different frequencies, but the frequency at which it resonates at any particular time is influenced by the resonance it picks up from its neighbors. In other words, each cell in the resonating system is a self-amplifying resonator, with a natural tendency to lock into phase with the resonance in the tissue around it. In fact, the tissue of the cardiac muscle has been shown to exhibit exactly this kind of spontaneous resonance. When individual cells of the cardiac muscle are separated from the bulk muscle and maintained in isolation *in vitro*, they are observed to oscillate electrically, each at its own natural frequency, but when they are assembled into bulk tissue, even if only by contact, they automatically adapt to each other's oscillations, to produce a single synchronized oscillation of the bulk muscle as a whole. As early as 1953 Bremer (1953) observed spontaneous electrical oscillations of the cat spinal cord that maintain synchronization from one end of the cord to the other, even when the cord is severed and reconnected by contact alone, whereas when completely separated, each fragment oscillates independently. A similar phenomenon is observed when a live snake is chopped into segments, each segment continues to writhe periodically for some time before it eventually dies, whereas in the whole snake the different parts all writhe in unison with a global wave pattern. A similar kind of synchrony between independent oscillators is also observed in the flashing lights of fireflies. Each individual firefly flashes at their own particular frequency when kept in isolation, whereas when released in the open, huge swarms of fireflies all flash in synchrony sometimes with tens of thousands of other individuals. And a similar effect is observed in the chirping of crickets, that also synchronize their chirps with each other. I propose that the principle that synchronizes fireflies and crickets to flash or chirp in synchrony, is the same principle that synchronizes the various parts of a firefly's brain so as to behave as an integrated whole, a unitary Gestalt, as also observed in visuospatial

experience, and in the dynamic symmetry and balance of motor patterns across space and time. This is the emergent self-organizing pattern formation principle exploited by nature for its templates for geometrical forms. The resonance automatically discovers or “computes” the array of harmonics of the resonating system because the resonances feed back on, or amplify themselves, and thus they occur more readily, at a lower energy, than non-harmonic vibrations that tend to cancel each other by destructive interference.

Falstad’s box modes applet, and the various examples of resonance described above, represent acoustical standing waves composed of patterns of higher and lower instantaneous air pressure. But the same patterns of standing wave resonance will appear in any volumetric resonating system that has the most general properties required of a harmonic resonant continuum, from acoustical vibrations in a hollow box, to mechanical vibrations in a solid body, to electromagnetic oscillations in a semiconductor crystal, to standing waves of laser light in a lasing optical cavity, or microwaves in a maser cavity, and of course, electrochemical oscillations in neural tissue that project spatial patterns across the brain and nervous system. All of these diverse systems can be contrived to produce volumetric spatial patterns as in Figure 2.5, and each pattern is associated with a specific temporal frequency of oscillation, a musical pitch, or tone, in the case of acoustical vibrations. The frequencies of the harmonic modes of an acoustical box are therefore related to each other harmonically, like the notes in a musical scale. It is the relation between the frequency of a vibration, and the spatial pattern of its standing wave, that allows a frequency encoding of spatial patterns in the brain.

### **Temporal Frequency Encoding**

Falstad’s Box Modes Applet displays a row of grids of little squares under the box modes simulation, representing the various harmonics of the box, organized in  $[x,y]$  grids, and the  $z$  dimension is represented by different layers of  $[x,y]$  grids from left to right. The first  $[x,y]$  grid on the left represents the  $x$  and  $y$  modes with  $z = 0$ ; the second grid shows the  $[x,y]$  modes with  $z = 1$ , the third grid represents  $z = 2$ , and so forth to higher harmonics of  $z$  in successive planes of  $[x,y]$  grids, representing a cubical  $[x,y,z]$  array. These squares can be clicked in Falstad’s simulation to make the corresponding standing wave patterns appear in the simulation, as shown in Figure 2.7. For example clicking the square  $[1,0,0]$  turns on the first harmonic standing wave in  $x$ , as shown in Figure 2.7 A, while clicking  $[0,1,0]$  turns on the first harmonic in  $y$ , as shown in Figure 2.7 B. Clicking the

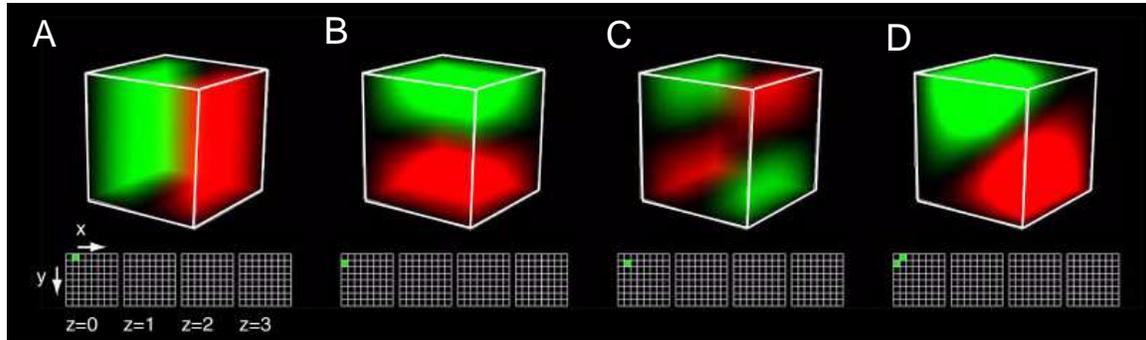


Figure 2.7. Harmonic frequency representation of the standing waves in the Box Modes applet, showing A: First harmonic in x, or  $[x,y,z]$  of  $[1, 0, 0]$ . B: First harmonic in y, or  $[0, 1, 0]$ . C: Cross product of first harmonics in x and y, or  $[1,1,0]$ . D: Simultaneous first harmonics in x and y, or  $[1,0,0]$  and  $[0,1,0]$ .

square  $[1,1,0]$  turns on the cross-product of those two harmonics, as shown in Figure 2.7 C. The harmonics can also be combined, by turning on  $[1,0,0]$  and  $[0,1,0]$  both at the same time, as shown in Figure 2.7 D. Note how the combined harmonics in Figure 2.7 D represents an *additive* type of function, or summation of the component patterns; that is, the compound pattern is positive in the quadrant where the sum of x and y components is positive, and negative where the sum is negative. The cross product in Figure 2.7 C, on the other hand, represents a *multiplicative* or *conjunctive* combination of the components, that is, the polarity reversal in each dimension changes the polarity of the pattern in the other dimension also, producing two polarity reversals, one across x, and the other across y, with zero-valued nodes located wherever either the x or the y pattern is zero. The standing waves in the resonant cavity therefore express a unique kind of *spatial logic*, in which the phase of vibration at any point in the volume is determined by a logical AND or logical OR type function on the component harmonics, expressed across extended spatial fields.

There is an interesting *dimensional reduction* in this representation of spatial waveforms by temporal frequencies, which serves as the basis for the principle of symbolic abstraction in biological computation. Imagine that the grids of little squares in Falstad's Box Modes applet are like electronic keyboards, that each produce a single musical note for each key (little square) when pressed (clicked) individually, or harmonious chords when pressed in combination. The keys on these keyboards define a musical scale. For example if key  $[1,0,0]$ , the fundamental, is the note C, then key  $[2,0,0]$  would be higher by an interval of a fifth, or G,  $2/3$  of the fundamental frequency; key  $[3,0,0]$ , double the fundamental frequency, would be C an octave above the fundamental; key  $[4,0,0]$  would be a frequency of  $5/4$  of the fundamental, or E, and so forth up the scale. Each note in

this harmonious progression corresponds directly to one of the waveforms in the array of standing waves, while harmonious chords produced by playing multiple notes simultaneously correspond to the compound patterns composed of combinations of those harmonic components.

Analog electronic keyboards, like their acoustical counterparts in pianos, harpsicords, and organs, produce their musical notes by a simple one-dimensional resonance in an appropriate resonator: a tuned circuit, a stretched string, or a tuned pipe. A resonance can be established in a resonator either “top-down”, for example by striking or plucking the string with a hammer or pick, or it can be produced “bottom-up”, by simply playing the note to which the resonator is tuned, and thus stimulating a sympathetic resonance in the resonator. For example, imagine a rank of organ pipes adjacent to the acoustical box, as depicted in Figure 2.6 C, whose pipes are tuned specifically to the harmonics of the acoustical box. If a particular standing wave is vibrating in the acoustical box, the vibration will stimulate a sympathetic vibration in the organ pipe whose frequency matches that of the resonance, (especially if amplification is provided at some point in the feedback loop) thus the presence of that particular standing wave pattern in the box is registered or “detected” by the resonance that emerges in the corresponding organ pipe, in the same way that a matching pattern in the receptive field of a neuron in a neural network model stimulates activation in its cell body. The simple one-dimensional resonances in the organ pipes are a *dimensionally reduced*, or *abstracted* symbolic representation of the corresponding volumetric spatial pattern that appears in the box, like a warning light on an instrument panel that registers some vital state of the system to which it is connected, in highly reduced symbolic form. In the case of the resonance model, the relationship between the spatial pattern in the box and its reduced symbolic representation in the organ pipe, is established not just by definition, or by connection, but by the fact that the pattern in the box evokes activation or resonance in the corresponding tube as a “bottom up” recognition of that pattern, and also by that fact that “top down” activation of the organ pipe by blowing air through it, automatically generates a reified exemplar of the corresponding waveform in the acoustical box in Figure 2.6. The extended spatial pattern and its reduced symbolic abstraction are thus intimately coupled in a bi-directional causal relationship in which the presence of either one immediately stimulates a manifestation of the other, even though they are expressed in complementary representational codes. This goes significantly beyond the computational paradigm of the neural network concept of neural activation triggered by patterns

in its receptive field, because the bottom-up mechanism behind the abstract recognition automatically includes also a top-down reification of the symbolic abstraction, as an essential aspect of the recognition mechanism. There is no need to define additional top-down neurons with appropriately patterned projective fields, as is often proposed in feedback neural network models, because the feedback through reciprocal interactions is already part of the principle of recognition. And it is not just a simple feedback connection, but an actual forward-and-inverse transformation that expands or reifies the dimensionally reduced symbolic code back into the explicit spatial pattern that it represents by an emergent Gestalt-like process that is characteristic of harmonic resonance.

In the next chapter we will explore the properties of harmonic resonance that make it so useful as a computational and representational principle in biological computation.