Smith's Left-Hand Rule

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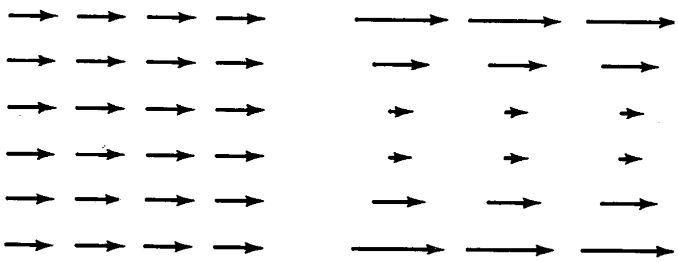
1. Introduction

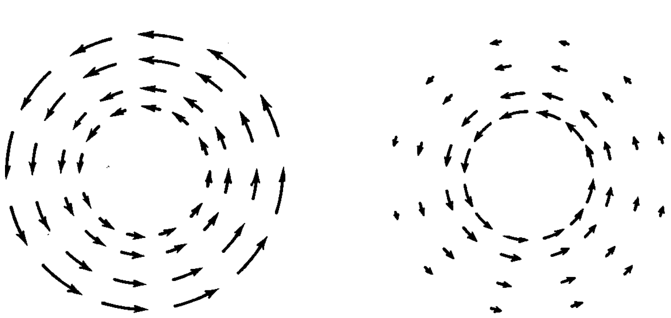
Many people who have had a scientific education are familiar with Fleming's Left-Hand (LH) and Right-Hand (RH) rules. Those who are unfamiliar with vector math do not realize that this is a simple way to describe vector multiplication for the special case where the two vectors being multiplied are at right angles to each other, with the resultant vector then being at right angle to both of those. These three vectors all at right angles to each other are easily visualised by the first and second fingers and the thumb of the hand. Fleming's rules apply to current flowing along a straight wire (one vector) while within a magnetic field (another vector) and the wire is moving in a certain direction (a third vector). These simple rules have allowed generations to understand how electric motors (LH rule) and generators (RH rule) work.

In this paper we introduce Smith's LH rule that applies to another form of vector math, the vector operator known as the Curl. In the science of electromagnetics a magnetic field (usually given the symbol **B**) is the result of the Curl of the magnetic vector potential, which is another field (usually given the symbol **A**). That **A** field has important properties that are generally not well known or understood, hence it isn't well taught to beginners and is something of a mystery to most experimenters looking into magnetic systems.

The Curl operator is often taught as the tendency for a vector to curl around on itself, hence the name **Curl** (in some languages the symbol **Rot** is used to imply a rotation). That wrongly implies we are dealing with vector fields that describe a vortex, that the Curl describes a property where the vector tends to curl, or change direction as you move along the vector direction. *That is not the property of the Curl operator.* Although the Curl operator can apply to vortex fields where the vector does curl, in fact vector fields that remain as straight lines can also have Curl. The following figure (taken from "Vector Fields" by Warren C Boast) shows some straight-line fields and some vortex fields that have or do not have Curl.



(a) Vector field with no curl   (b) Vector field with curl



(c) Vector field with curl   (d) Vector field with no curl

The Curl operator is better described as the property of a vector field where the vector changes its amplitude with distance, not taken along the vector direction but *taken at right angles to the vector* . The result of the Curl operation on vector **A** yields another vector that is at right angles to both **A** itself and the distance direction taken at right angles to **A.** Thus if **A** has a specific changing of amplitude along that sideways distance, that final vector is proportional to that change and is the magnetic field **B**. Hence these three vectors (**A**, **B** and **Sideways** movement) can also easily be visualised by the first and second fingers and the thumb of the hand. Note that not all vectors that change amplitude with sideways distance produce a Curl result, see for example (d) in the previous figure. However this LH Rule can be useful for either envisaging the **B** field that results from a known **A** field, or envisaging the **A** field that produces a known **B** field.

2. Smith's LH Rule

The figure below shows the rule as applied to the left hand. The following guides allow anyone to understand which finger applies to each vector.

- **F**irst finger. The letter **F** is for magnetic vector potential **F**ield. The **F**irst letter of the alphabet is **A**, this is the **A** field.

- **S**e**C**ond finger. The letter **C** is for **C**hange, this is the **S**ideways direction in which the magnetic vector potential **A** field is **C**hanging. It is the direction in which the field amplitude is increasing, which can be towards a distant **S**ource.

- **T**hum**B**. This is the direction of the resultant magnetic **B** field. The international unit for the **B** field is **T**esla.