

# Gyrator–Capacitor Modeling: A Better Way of Understanding Magnetic Components

David C. Hamill

Department of Electronic and Electrical Engineering  
University of Surrey  
Guildford GU2 5XH, United Kingdom

**Abstract** — Engineers new to power electronics often find magnetic components puzzling, especially multi-winding devices and integrated magnetics. Gyrator–capacitor modeling offers an alternative way of understanding such components. This tutorial paper begins with a review of the gyrator–capacitor approach, followed by some examples drawn from the field of dc-dc conversion: a simple choke, a flyback-converter transformer, a pair of coupled chokes — in which the zero-ripple phenomenon is explained — and a two-output forward-converter transformer. Finally, core saturation is modeled by means of nonlinear capacitance and illustrated by SPICE simulation. Gyrator–capacitor modeling offers a unified, logical way of understanding the magnetic components commonly met with in power electronics.

## I. INTRODUCTION

Engineers new to power electronics often find magnetic components difficult to understand, especially multi-winding devices such as coupled chokes and integrated transformer–choke assemblies. There seem to be several reasons:

- Inductors are not commonly found in ordinary electronics. In low-frequency analog circuitry they have largely been superseded by capacitance combined with high-gain amplifiers; power electronics and radio-frequency electronics are the exceptions.
- The imperfections of inductive components are considerable: winding loss, core loss, nonlinearities and temperature variation must all be taken into account.
- Inductors are not usually available off-the-shelf, but must be designed and made specially. (Having to make one's own capacitors would reduce their popularity!)
- Multi-port devices are more complex than simple impedances. A linear impedance is characterized by a single function, whereas a linear  $N$ -port requires a matrix of  $N^2$  functions.

Capacitors do not suffer from the same drawbacks: they are common throughout electronics, they are closer to ideal than inductors, a gamut of types and specifications is readily

available, and they are inherently two-terminal devices. Moreover, the charge stored in a capacitor is readily visualized as 'a bunch of electrons,' whereas the flux-linkages of an inductor are more mysterious. Finally, the state variable associated with a capacitor is voltage, which is easily monitored by probing the component's terminals, whereas inductor current is not easily measured without breaking the circuit and inserting a resistor or a current transformer (itself a poorly understood magnetic device). All these factors conspire to make capacitors much more 'friendly' than inductors.

In an attempt to dispel some of the mystery surrounding inductive components, this tutorial paper begins with a review of gyrator–capacitor modeling. Then some examples are presented to show how it can aid the understanding of complex magnetic components. The method is also newly extended to include transformer core saturation.

## II. GYRATOR–CAPACITOR MODELING

In traditional equivalent-circuit models, e.g. [1], [2], magnetomotive force (mmf) is analogous to voltage, and magnetic flux is analogous to current; consequently, magnetic reluctance is represented by resistance. (It is confusing that a magnetic core, which stores energy, is represented by a network of resistors, which dissipate energy.) Moreover, if this resistance model is to be combined with an external electrical circuit, it must first be transformed into an inductance model. The process involves taking the dual of the resistance network, and replacing resistances by inductances. The topology of the resulting equivalent circuit bears little resemblance to that of the component it models, hiding the underlying relationships. For example, flux leakage, which is associated with *parallel* flux paths, causes *series* inductances in the equivalent circuit. This can be puzzling, especially where multi-winding components are concerned. If nonlinear core materials are taken into consideration, the traditional model becomes even more confusing. Thus, for example, Prof. Hammond [3] said he "has spent many years working on magnetic problems and finds the equivalent circuit ... a hindrance rather than a help."

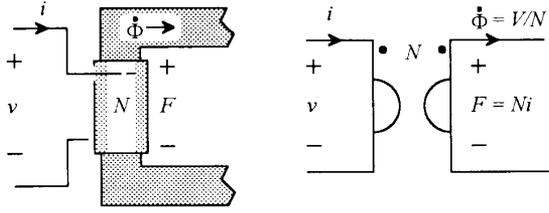


Fig. 1: A winding can be modeled by a gyrator.

The gyrator–capacitor method, originally introduced by R.W. Buntenbach about 25 years ago [4]–[6] and only recently revived [7], offers an alternative way of forming equivalent circuits of magnetic components. It seems clearer than the traditional method, and should help the student and practicing engineer alike to understand the operation of many different types of wound components.

In the gyrator–capacitor (G–C) approach, mmf  $F$  is again analogous to voltage, but current now corresponds to the *rate-of-change* of magnetic flux,  $d\Phi/dt$  or  $\dot{\Phi}$ . This quantity, which has been called the *flux-rate*, is expressed in units of webers per second. For a winding of  $N$  turns:

$$v = N \dot{\Phi} \quad (1)$$

$$i = F/N \quad (2)$$

The first is Faraday’s law; the second comes from the definition of mmf,  $F \equiv Ni$ . The left-hand sides of the equations refer to the electrical domain, while the right-hand sides refer to the magnetic domain. These equations have an interesting property: in (1) voltage is proportional to flux-rate, which is analogous to current; and in (2) current is proportional to mmf, which is analogous to voltage. So a winding has two main characteristics:

- It forms a bridge between the electrical domain and the magnetic domain.
- It converts a voltage into a current-like quantity (flux-rate), and converts a current into a voltage-like quantity (mmf).

Now, the gyrator is a two-port electrical circuit element that has a similar property of interchanging voltage and current. Its governing equations are:

$$v_1 = r i_2 \quad (3)$$

$$i_1 = v_2/r \quad (4)$$

where  $r$  is known as the gyration resistance. If, in an equivalent circuit, flux-rate is represented by current and mmf by voltage, then a comparison of equations (1)–(4) shows that

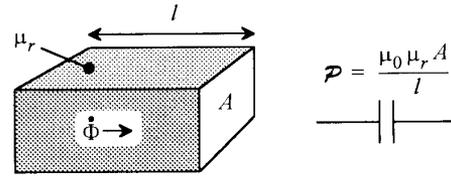


Fig. 2: The permeance of a magnetic path can be modeled by a capacitor.

an  $N$ -turn winding can be properly represented by an  $N$ -Ω gyrator. The sign conventions are illustrated in Fig. 1.

The other thing to be modeled is the magnetic circuit itself, which comprises a set of flux paths through ferrite, air, etc. Conventionally each path element is represented by a *reluctance*, but it is more convenient here to use *permeance*, its reciprocal. This is because, when flux-rate and mmf are the analogs of current and voltage respectively, magnetic permeance corresponds to capacitance. The value of the capacitance is given by

$$\mathcal{P} = \frac{\mu_0 \mu_r A}{l} \quad (5)$$

where  $\mu_0 = 4\pi \times 10^{-7}$ ,  $\mu_r$  is the magnetic permeability of the material,  $A$  is the cross-sectional area of the path and  $l$  is the path length, as illustrated in Fig. 2. Flux-rate and mmf in this ‘capacitor’ are linked by the relation

$$\dot{\Phi} = \mathcal{P} \frac{dF}{dt} \quad (6)$$

Then a magnetic core is represented by a capacitive network with a similar topological structure; the capacitance values can be calculated from the geometry. (In practice it may be difficult to find the effective permeance of a path through air, as the flux distribution is not easily determined.)

To summarize, in G–C modeling the form of the equivalent circuit follows the physical form of the component, with permeances shown as capacitors and windings represented by gyrators in the appropriate locations. In most cases the form of the G–C model can be obtained by inspection. The G–C approach has some clear-cut advantages over traditional modeling:

- Energy relationships are preserved. Both the quantity of energy and its distribution within the component are correctly represented.
- There are no difficulties with non-planar magnetic structures or other special cases.
- Because the equivalent circuit is topologically similar to the magnetic component, it is easier to understand how

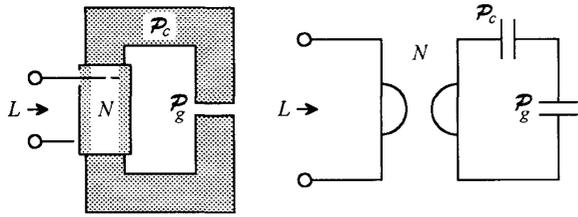


Fig. 3: A simple choke with a gapped core, and its gyrator-capacitor model.

the component operates, and how it interacts with the external circuit.

- Simulation with SPICE and similar programs presents few problems [7].

### III. BUT WHAT IS A GYRATOR?

The main obstacle to understanding the G-C approach is the notion of a gyrator. Briefly, a gyrator is an ideal two-port circuit element that reflects an impedance at one port as its reciprocal at the other. In other words, it takes the dual of the impedance — perhaps a better name would be a ‘dualizer.’ For example, a capacitance at one port is reflected as an inductance at the other. Once this property is appreciated, there is only one further thing to learn: the gyrator has a numerical scaling factor of  $r^2$ , where  $r$  is the *gyration resistance*. So a 1- $\mu\text{F}$  capacitor connected to one port of a 5- $\Omega$  gyrator is reflected at the other port as an inductance of  $5^2 \times 10^{-6} = 25 \mu\text{H}$  — and vice versa. Similarly, a voltage source is reflected as a current source, and a short circuit as an open circuit, and vice versa.

The gyrator should be no more difficult to understand than the ideal transformer, a two-port circuit element that is taught to undergraduates. It, too, has a numerical transformation ratio,  $n$ . It, too, reflects an impedance at one port as a different impedance at the other port. The governing equations are quite similar for the two devices:

$$\begin{array}{ll}
 v_1 = r i_2 & v_1 = n v_2 \\
 i_1 = v_2/r & i_1 = i_2/n
 \end{array} \quad (7)$$

(ideal gyrator)                      (ideal transformer)

### IV. EXAMPLES

To illustrate how G-C modeling may be applied, a series of examples is presented next, drawn from the field of dc-dc conversion: a simple choke, a flyback-converter transformer, a pair of coupled chokes, and a two-output forward-converter

transformer. Finally, the effect of core saturation is examined.

#### A. Simple Choke

Fig. 3 shows an inductor with a gapped core, an assembly often found as a dc smoothing choke. The core and gap permeances are represented by capacitors, and a winding of  $N$  turns is represented by a gyrator of  $N \Omega$ . Now it is often stated that in such a choke, most of the energy is stored in the airgap. Why is this? The mmf  $F = Ni$  (analogous to voltage) divides between the gap and core permeances  $\mathcal{P}_g$  and  $\mathcal{P}_c$  (analogous to capacitances). Because the core material has a high relative permeability,  $\mu_r \sim 2000$  for a typical ferrite,  $\mathcal{P}_g$  is much smaller than  $\mathcal{P}_c$  even though the magnetic path length through the core is much longer than that through the gap. Thinking of the permeances as capacitances,  $\mathcal{P}_g$  has a much higher impedance than  $\mathcal{P}_c$ , so most of the mmf (‘voltage’) appears across the gap. Since the energy in a capacitor is proportional to voltage squared, a simple calculation shows that the energy divides so that  $\mathcal{P}_c/(\mathcal{P}_c + \mathcal{P}_g)$  of the total — i.e. most of it — is stored in the gap.

Again treating the permeances like capacitors in series, and applying the ‘dualizer’ principle, the inductance of the choke can be found to be  $L = N^2/(1/\mathcal{P}_c + 1/\mathcal{P}_g) \approx N^2\mathcal{P}_g$ .

In the periodic steady state, the average voltage across the choke must be zero. The periodic steady state is characterized by the fact that the conditions at the beginning and end of each cycle are identical. In particular, the flux  $\Phi$  must be the same at the beginning and end of each cycle. Thus, averaged over a cycle, its rate-of-change (the flux-rate  $\dot{\Phi}$ ) must be zero:  $\langle \dot{\Phi} \rangle = 0$ , where angle brackets  $\langle \rangle$  denote an average. Since  $\dot{\Phi} = v/N$ ,  $\langle v \rangle = 0$  too: the average voltage across the choke winding must be zero.

However, the same does not apply for current: the choke winding can pass a dc, which merely applies a constant ‘bias voltage’ to ‘capacitors’  $\mathcal{P}_g$  and  $\mathcal{P}_c$ . As long as this ‘bias voltage’ is not too great, ‘capacitor’  $\mathcal{P}_c$  will not alter its value (saturate). This effect will be dealt with in detail below.

#### B. Flyback-Converter Transformer

The transformer found in the isolated flyback dc-dc converter is essentially a choke with two windings. The primary winding absorbs energy from the input circuit and stores it mostly in the gap of the core assembly; the secondary extracts the energy and feeds it into the output circuit. However, the interaction between the magnetic circuit and the external electrical circuit is not easily fathomed. If the G-C equivalent-circuit model of the transformer is drawn and combined with the external electrical circuit, the operation of

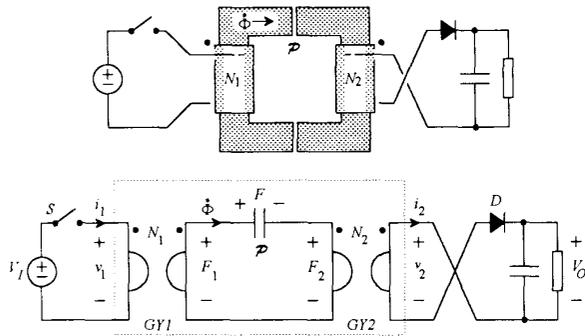


Fig. 4: The isolated flyback converter, and an equivalent circuit incorporating a gyrator-capacitor model of the transformer.

the circuit can be determined by a logical, step-by-step process.

Here, for simplicity, leakage is neglected and the circuit is assumed to operate in discontinuous mode. The core and gap permeances, which appear in series, are combined into a single permeance  $\mathcal{P}$ . With reference to the circuit of Fig. 4, the argument is as follows.

#### i. $S$ Closed

In the first phase of operation, with switch  $S$  closed, the input voltage  $V_1$  appears across the primary winding of  $N_1$  turns, represented by gyrator  $GY1$ . Working from left to right:

- 1) The flux-rate is  $\dot{\Phi} = v_1/N_1 = V_1/N_1$ , and this is constant. (The flux increases linearly with time.) Note that  $GY1$  effectively translates the voltage source  $V_1$  into a 'current' source,  $\dot{\Phi}$ .
- 2) For permeance  $\mathcal{P}$ , the flux-rate-mmf relation can be applied to find  $dF/dt = \dot{\Phi}/\mathcal{P} = V_1/N_1\mathcal{P}$ . Again this is constant, so  $F$  ramps up linearly, starting from zero and following  $F = (V_1/N_1\mathcal{P})t$ . Thus as 'capacitance'  $\mathcal{P}$  'charges', its 'voltage'  $F$  increases linearly with time.
- 3) Since there is no leakage, the same  $\dot{\Phi}$  flows through the secondary winding of  $N_2$  turns, represented by gyrator  $GY2$ . The secondary voltage is found as  $v_2 = N_2\dot{\Phi} = (N_2/N_1)V_1$ . Thus the primary voltage appears at the secondary, transformed by the turns ratio.
- 4) Applying KVL to the secondary circuit, diode  $D$  is reverse-biased by a voltage  $V_o + (N_2/N_1)V_1$ . Therefore the diode must be in its blocking state, with  $i_2 = 0$ .

How does this state reflect back to the primary circuit? Working from right to left:

5) At  $GY2$ ,  $i_2 = 0$  means that  $F_2 = N_2i_2 = 0$ . By KVL,  $F_1 = F = (V_1/N_1\mathcal{P})t$ .

6) Now at  $GY1$ ,  $i_1 = F_1/N_1 = (V_1/N_1^2\mathcal{P})t$ . Therefore the primary current ramps up linearly.

This process continues until the switch is opened at time  $t_1$ , at which point  $i_1$  has attained a value of  $I_{1(pk)} = (V_1/N_1^2\mathcal{P})t_1$ .

#### ii. $S$ Opens at $t = t_1$

In the second phase of operation, switch  $S$  is open. Working from left to right:

- 1) Since  $S$  is open,  $i_1 = 0$ . At  $GY1$ ,  $F_1 = N_1i_1 = 0$  also.
- 2) By KVL,  $F_2 = -F = -(V_1/N_1\mathcal{P})t_1$  at the instant when  $S$  opens. At  $GY2$ ,  $i_2 = F_2/N_2 = -(V_1/N_1N_2\mathcal{P})t_1$ . This is equal to  $-(N_1/N_2)I_{1(pk)}$ , the peak primary current transformed by the turns ratio.
- 3) Since  $i_2 < 0$ ,  $D$  is forward-biased. The output voltage appears across the secondary winding:  $v_2 = -V_o$ .

This voltage now reflects back to the primary. Working from right to left:

- 4) At  $GY2$ ,  $\dot{\Phi} = v_2/N_2 = -V_o/N_2$ .
- 5) For permeance  $\mathcal{P}$ , the flux-rate-mmf relation gives  $dF/dt = \dot{\Phi}/\mathcal{P} = -V_o/N_2\mathcal{P}$ .
- 6) At  $GY1$ ,  $v_1 = N_1\dot{\Phi} = -(N_1/N_2)V_o$ . The output voltage is reflected back to the primary winding, modified by the turns ratio. Applying KVL to the input circuit,  $S$  sees a voltage of  $V_1 + (N_1/N_2)V_o$ .

This argument is logical and quite unlike conventional descriptions of the flyback converter and its transformer. As a rule, these resort to ideas such as 'collapsing flux', 'clamped back-emf', and 'ampere-turns balance,' which students often find difficult to grasp.

#### C. Coupled Chokes

The motivation for using coupled smoothing chokes in a multi-output converter is threefold:

- To save mass by sharing a single core between two or more chokes.
- To improve dynamic cross regulation by increasing the cross-coupling among outputs.
- To take advantage of the 'zero-ripple' phenomenon, whereby the ripple in one winding can be reduced to zero (at the expense of increased ripple in the others).

It is the last that interests us here. The zero-ripple phenomenon can be explained by the G-C model of the coupled-choke assembly, which is similar to that of the flyback

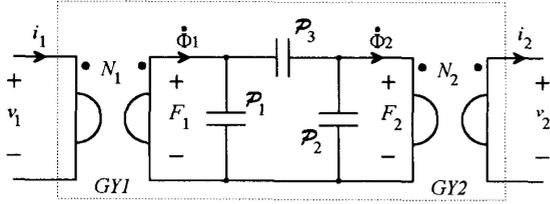


Fig. 5: Equivalent circuit of a pair of coupled chokes.

transformer. The difference lies in the way the component interacts with the external electrical circuit. Consider the equivalent circuit of Fig. 5, which includes permeances  $\mathcal{P}_1$  and  $\mathcal{P}_2$  to represent the leakage paths associated with windings  $N_1$  and  $N_2$ . The leakage can be characterized by a coupling coefficient,  $k$ , defined as the proportion of flux (and hence flux-rate) produced by winding  $N_1$  that reaches winding  $N_2$ :

$$k = \frac{\dot{\Phi}_2(t)}{\dot{\Phi}_1(t)} \quad (8)$$

Voltages  $v_1$  and  $v_2$  are arbitrary functions of time, but to avoid flux ramping they must have an average value of zero, as for any choke. Because  $\dot{\Phi} = v/N$ , the flux-rates also have a zero average. Currents  $i_1$  and  $i_2$  are arbitrary functions of time, and comprise an ac (ripple) term plus a dc term; however, for the time being, let us assume that the dc terms are zero. Our objective is to find the condition that eliminates one of the ripple currents: to make  $i_2(t) = 0$ , say.

For gyrator  $GY2$ , if  $i_2(t) = 0$  then  $F_2(t) = N_2 i_2 = 0$ . In other words, the right-hand end of  $\mathcal{P}_3$  is grounded. The flux-rate generated by winding  $N_1$ ,  $\dot{\Phi}_1(t)$ , splits between  $\mathcal{P}_1$  and  $\mathcal{P}_3$ , which form a ‘current’ divider. Elementary circuit theory tells us that  $\dot{\Phi}_2/\dot{\Phi}_1 = \mathcal{P}_3/(\mathcal{P}_1 + \mathcal{P}_3) = k$ . Applying  $\dot{\Phi} = v/N$  to each gyrator, we find:

$$\frac{v_2(t)}{v_1(t)} = k \cdot \frac{N_2}{N_1} \quad (9)$$

Equation (9) is the zero-ripple condition for  $i_2$ . Since the right-hand side of the equation is constant, the two voltages must have the same waveform. If the ratio of their amplitudes is fixed by circuit operation, then zero ripple in  $i_2$  is obtained by choosing appropriate values for  $k$ ,  $N_1$  and  $N_2$ .

If we repeat the above argument but this time aiming to make  $i_1(t) = 0$ , we find the new zero-ripple condition to be

$$\frac{v_1(t)}{v_2(t)} = k' \cdot \frac{N_1}{N_2} \quad (10)$$

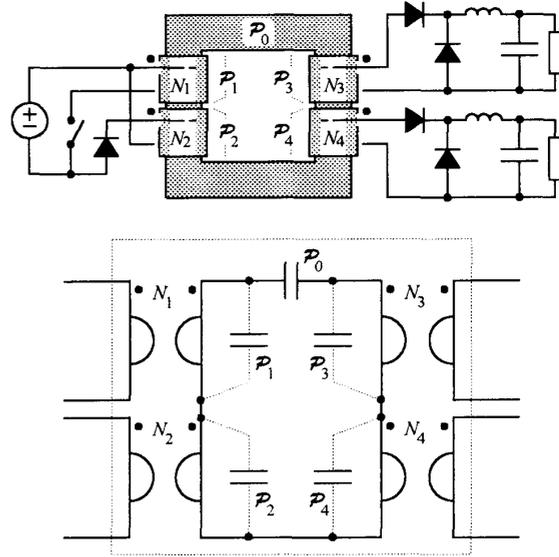


Fig. 6: Two-output forward converter, and a gyrator–capacitor model of its transformer, including leakage paths.

where  $\dot{\Phi}_1/\dot{\Phi}_2 = \mathcal{P}_3/(\mathcal{P}_2 + \mathcal{P}_3) = k'$ . In other words, by choosing the coupling appropriately we can arrange to have zero ripple current in either of the windings.

Is it possible to have zero ripple current in both windings simultaneously? No, because manipulating (9) and (10) we find we need  $k k' = 1$ ; this is impossible since both  $k$  and  $k'$  must be less than unity in any practical magnetic device, as there will always be some leakage.

Because the coupled-choke assembly is assumed to be completely linear, we can invoke the superposition principle to find the effect of the dc terms in  $i_1$  and  $i_2$ . Since  $F = Ni$ , the only effect of the dc is to apply a ‘bias voltage’ to the ‘capacitors.’ So the zero-ripple condition is independent of the dc currents, which are arbitrary. (In practice we must ensure that the currents are small enough to avoid core saturation, or the assumption of linearity will be invalidated.)

#### D. Forward-Converter Transformer

Fig. 6 shows a two-output forward converter whose transformer incorporates leakage paths. The core permeance has been lumped as  $\mathcal{P}_0$ , with leakage paths  $\mathcal{P}_1$ – $\mathcal{P}_4$  added for windings  $N_1$ – $N_4$  respectively. (This is a rather unlikely arrangement; a more practical design might employ a pair of E cores. The object here is to show the modeling principles while avoiding unnecessary detail.) A G–C model of the transformer is also shown.

Assuming no leakage, the operation of the forward-converter transformer can be explained using arguments

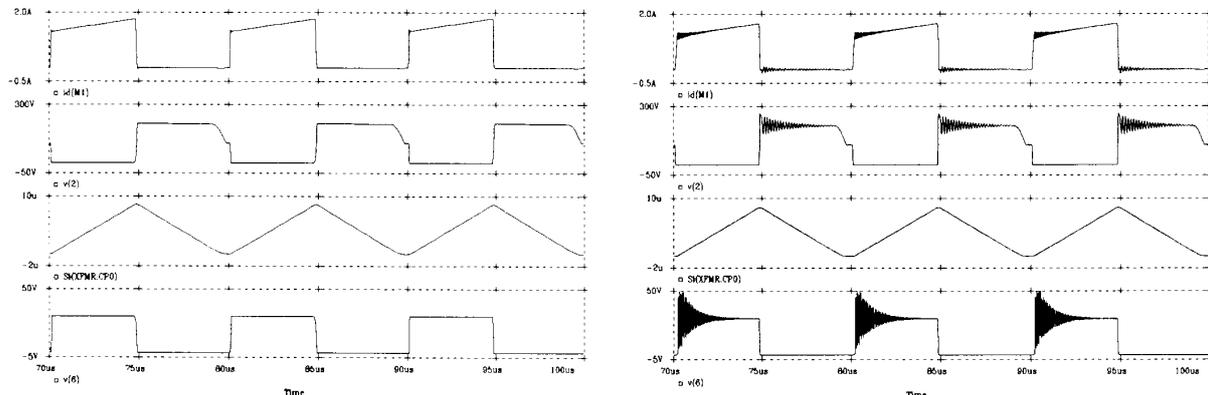


Fig. 7: Forward converter waveforms, simulated with PSpice. The left-hand set is with tight coupling of all windings to the core ( $\mathcal{P}_1$ – $\mathcal{P}_4$  zero), while the right-hand set is with leakage permeances included ( $\mathcal{P}_1$ – $\mathcal{P}_4$  set to  $2\text{nH} \Leftrightarrow 2\text{nF}$ ). Top to bottom: switch current, switch voltage, core flux [ $\mu\text{Wb}$ ], voltage at the input to the 12-V output smoothing choke (other channel is similar).

similar to those for the flyback transformer. With the switch closed, the input voltage  $V_1$  appears across primary winding  $N_1$ , causing a flux-rate  $\dot{\Phi} = v_1/N_1 = V_1/N_1$  to flow around the magnetic circuit. A voltage  $v_2 = N_2\dot{\Phi} = V_1 N_2/N_1$  is induced across winding  $N_2$ , and similarly for the secondary windings,  $N_3$  and  $N_4$ . Because of the phasing of the windings and the polarity of their associated diodes, the two secondary windings conduct but winding  $N_2$  does not. When the switch opens, a similar argument to that employed for the flyback converter shows that only the energy-recovery winding  $N_2$  conducts, with voltage  $-V_1$  across it, giving a negative flux-rate  $\dot{\Phi} = v_2/N_2 = -V_1/N_1$ . Thus the ‘charge’ on ‘capacitor’  $\mathcal{P}_0$  decreases linearly until it reaches zero, at which time the core reaches a demagnetized state (i.e., zero flux).

This is demonstrated by the PSpice simulation results of Fig. 7. The circuit parameters are as follows: input voltage 100V, output voltages 12V and 5V, switching frequency 100kHz, and duty factor 0.45. The models employed for the switch (a MOSFET) and the diodes are intended to be realistic, incorporating nonlinear capacitive effects. The transformer core has  $A = 35\text{mm}^2$ ,  $l = 80\text{mm}$  and  $\mu_r = 1500$ ; from (5) its permeance is  $\mathcal{P}_0 = 825\text{nH} (\Leftrightarrow 825\text{nF})$ . The windings have the following numbers of turns:  $N_1 = N_2 = 55$ ,  $N_3 = 16$ ,  $N_4 = 7$  ( $\Leftrightarrow 55\Omega$ ,  $16\Omega$ ,  $7\Omega$ ). For the left-hand set of waveforms, there is no magnetic leakage, and the waveforms approach the ideal. The effect of leakage is demonstrated in the right-hand set of waveforms, where permeances  $\mathcal{P}_1$ – $\mathcal{P}_4$  are each made  $2\text{nH} (\Leftrightarrow 2\text{nF})$ , corresponding to 0.24% leakage. Some complicated resonances occur among the leakage inductances and parasitic capacitances. To investigate this further, each leakage permeance could be varied in turn.

Though not attempted here, the two smoothing chokes could be coupled and modeled by a G–C equivalent circuit.

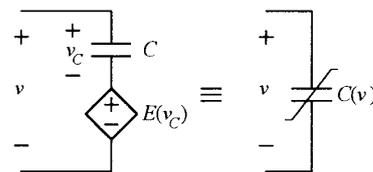


Fig. 8: A nonlinear capacitance can be emulated in SPICE by means of a linear capacitor and a voltage-controlled voltage source.

More details on simulating integrated magnetics are contained in [7].

### E. Core Saturation

At high mmf excursions, real magnetic materials exhibit saturation and hysteresis effects. The permeance of a core made of such a material is no longer constant, but depends on the applied mmf. To model an anhysteretic saturation characteristic, the capacitors representing a core can be given a nonlinear  $q$ – $v$  characteristic. In SPICE an appropriate shaping may be obtained by a method that is the dual of one devised by Pei and Lauritzen [8]: a linear capacitor is connected in series with a voltage-controlled voltage source, as shown in Fig. 8, and the source voltage is made an odd polynomial function of the capacitor voltage. In the simplest case this function is

$$E(v_c) = a v_c^n \quad (11)$$

where  $E$  is the voltage of the controlled source,  $v_c$  is the voltage across capacitance  $C$ , and  $n$  is an odd integer (typically 11 for a ferrite). The effective capacitance of the combination is:

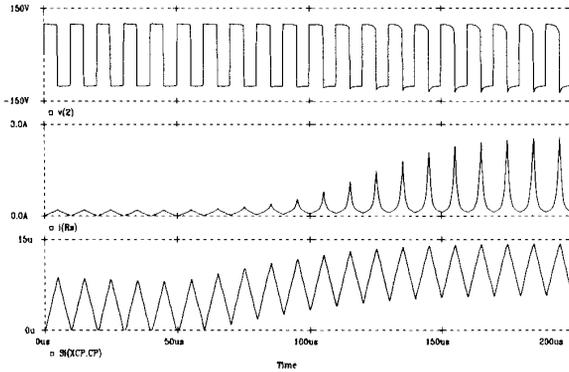


Fig. 9: PSpice simulation results for saturating forward-converter transformer. Top to bottom: primary voltage, primary current, core flux  $\Phi$  in  $\mathcal{P}_0$  [Wb].

$$C_{eff}(v) = \frac{C}{1 + an v^{n-1}} \quad (12)$$

where  $v$  is the applied voltage. At low voltages  $C_{eff} \rightarrow C$ , but at high voltages it reduces sharply at a rate determined by  $n$ . Parameter  $a$  controls the position of the knee of the curve. This model may be applied to each branch of a complex magnetic structure, allowing localized saturation to be investigated, within the limitations of lumped-parameter modeling.

As an example, this model is applied to the core of the forward-converter transformer treated above. For simplicity, only the primary winding  $N_1$  is retained. It is important to include the leakage permeance  $\mathcal{P}_1$  because, as the core starts to saturate, the leakage flux becomes a significant proportion of the total. (The leakage permeance remains linear, being a path through air.) The core permeance  $\mathcal{P}_0$  is modeled by a nonlinear capacitor with  $C = 825\text{nF}$  (cf. the linear case),  $n = 11$  and  $a = 3 \times 10^{-12}$ . A voltage pulse of  $\pm 100\text{V}$  and a frequency of  $100\text{kHz}$  is applied through a source resistance of  $10\Omega$ ; the duty factor is 0.50 for the first five cycles and 0.53 for the remainder, to induce flux ramping. Fig. 9 shows waveforms simulated with PSpice, while Fig. 10 shows the anhysteretic saturation characteristic traced out by the core permeance.

In principle, the core model can include hysteresis by using a more complicated nonlinear capacitor. This topic will be explored in a planned future paper.

## V. CONCLUSION

Gyrator-capacitor modeling is a simple, straightforward and logical way of modeling magnetic components. □

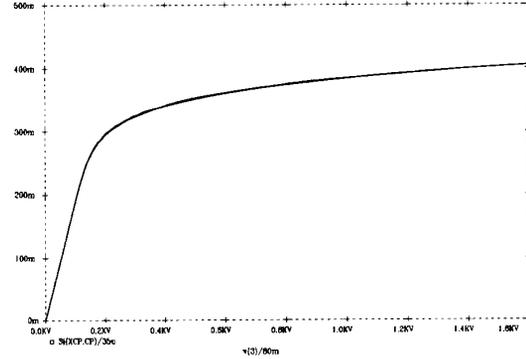


Fig. 10: Anhysteretic saturation  $B$ - $H$  curve for the nonlinear core. Vertical axis is  $B$  [T], horizontal axis is  $H$  [A/m].

Windings are represented by gyrators, and magnetic paths by capacitors. It is possible to model saturating core materials by nonlinear capacitors. The main obstacle seems to be understanding the principle of the gyrator, which can best be described as a 'dualizer.' The gyrator-capacitor method offers a unified way of understanding the magnetic components commonly used in power electronics.

## REFERENCES

- [1] S.A. El-Hamamsay and E.I. Chang, "Magnetics modeling for computer-aided design of power electronics circuits", *Power Electronics Specialists Conf.*, Milwaukee, WI., June 1989, vol. 2, pp. 635-645
- [2] A.F. Witulski, "Modeling and design of transformers and coupled inductors", *Applied Power Electronics Conf.*, San Diego, Ca., Mar. 1994, pp. 589-595
- [3] P. Hammond, *Electromagnetism for Engineers: An Introductory Course*, 1st ed., Oxford, U.K.: Pergamon Press, 1964
- [4] R.W. Buntentbach, "Improved circuit models for inductors wound on dissipative magnetic cores", *2nd Asilomar Conf. on Circuits & Systems*, Pacific Grove, Ca., Oct. 1968 (IEEE publ. no. 68C64-ASIL), pp. 229-236
- [5] R.W. Buntentbach, "Analogies between magnetic and electrical circuits", *Electronic Products*, vol. 12, no. 5, pp. 108-113, Oct. 1969
- [6] R.W. Buntentbach, "A generalized circuit model for multiwinding inductive devices" (digest only), *IEEE Trans. on Magnetics*, vol. 6, no. 1, p. 65, Mar. 1970
- [7] D.C. Hamill, "Lumped equivalent circuits of magnetic components: the gyrator-capacitor approach", *IEEE Trans. on Power Electronics*, vol. 8, no. 2, pp. 97-103, Apr. 1993
- [8] D. Pei and P.O. Lauritzen, "A computer model of magnetic saturation and hysteresis for use on SPICE2", *Power Electronics Specialists Conf.*, Gaithersburg, Md., June 1984, pp. 247-256