Evaluation and Measurement of Complex Permeability

Note1. There are two different symbols for representing imaginary numbers in common use, one using the symbol *i* while the other uses *j*, both have the significance of √(-1). Because of potential confusion with the use of *i* for current, the following paper uses the symbol *j*.

Note 2. The amplitude of a phasor can be expressed as a peak value, a peak-to-peak value, or a RMS value. In the following equations it does not matter providing that whichever is chosen is used consistently throughout.

Note 3. In the following paper use is made of the reluctance R_{AIR} of the *air space* occupied by the magnetic core material. Although this feature is commonly used for permanent magnets in order to establish a load-line, it is not generally used for soft materials. It has the advantage of separating out the relative permeability μ_R from the usual reluctance value thus making the equations more readily understandable.

1. On Complex Permeability

For a closed magnetic circuit (no air gaps, e.g. a ring core), the flux Φ is related to the mmf *Ni* and the reluctance *R* by magnetic Ohm's Law

$$\Phi = \frac{Ni}{R} \tag{1}$$

where *R* is given by

$$R = \frac{l}{\mu_R \mu_0 A} \tag{2}$$

A is the cross section area of the core, *l* its magnetic length and μ_R the relative permeability. It is convenient to make use of the reluctance R_{AIR} of the ***air space*** occupied by the core given by

$$R_{AIR} = \frac{l}{\mu_0 A} \tag{3}$$

so that we can express (1) in the form

$$\Phi = \frac{Ni}{R_{AIR}} \mu_R \tag{4}$$

1.1. Series Representation

At high frequencies a phase shift between *Ni* and Φ occurs which is accounted for by making μ_R complex

$$\mu_R = \mu_s' - j\mu_s'' \tag{5}$$

(the suffix *s* denotes the series feature of the equivalent LR circuit as seen below) so that (4) becomes

$$\Phi = \frac{Ni}{R_{AIR}}(\mu_s' - j\mu_s'') \tag{6}$$

This gives us the two components of Φ, $\dfrac{\mu_s' Ni}{R_{AIR}}$ being the flux that is in phase with the current while $\dfrac{\mu_s'' Ni}{R_{AIR}}$ is a component that is phase retarded by 90°, see figure 1.

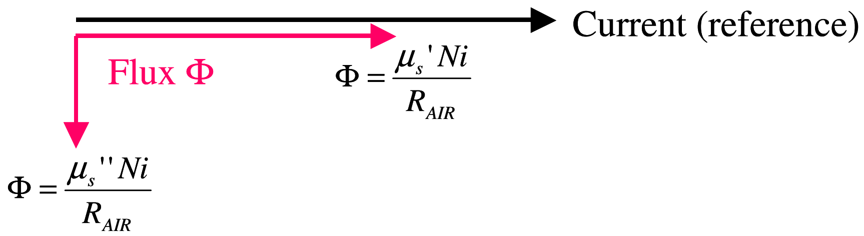

Figure 1. The two components of Flux

The voltage V across the coil is given by $V = N\frac{\mathrm{d}\Phi}{\mathrm{d}t}$. Differentiation involves multiplication by $j\omega$ (multiplication by j produces a 90° CCW rotation of the vector), hence from (6) we obtain

$$V = \frac{\omega N^2 i}{R_{AIR}}(\mu_s'' + j\mu_s') \qquad (7)$$

Thus we have

$$V_{LOSS} = \omega\mu_s''\frac{N^2 i}{R_{AIR}} \qquad (8)$$

as the voltage component that is in phase with the current. The ratio $\frac{V_{LOSS}}{i}$ represents a loss-resistance value $\mathrm{R} = \omega\mu_s''\frac{N^2}{R_{AIR}}$ in series with the inductance of the coil. We also have

$$V_{IND} = \omega\mu_s'\frac{N^2 i}{R_{AIR}} \qquad (9)$$

as the voltage component that is phase advanced by 90°. The ratio $\frac{V_{IND}}{\omega i}$ is of course the inductance value $\mathrm{L} = \mu_s'\frac{N^2}{R_{AIR}}$.

We see that the voltage measured across the coil is what we would get across a circuit consisting of L and R in series, figure2.

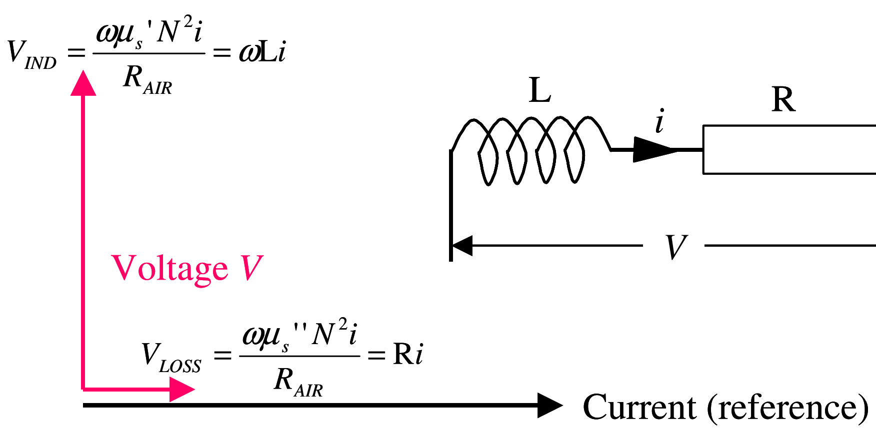

Figure 2. The two components of Voltage

1.2. Parallel Representation

There is an alternative version of complex permeability where the equivalent circuit LR is parallel. Here in place of (5) we have for μ_R

$$\frac{1}{\mu_R} = \frac{1}{\mu_p'} - \frac{1}{j\mu_p''} = \frac{1}{\mu_p'} + \frac{j}{\mu_p''} \qquad (10)$$

(Note the paper "Soft Ferrites and Accessories" referenced by Graham has an error in that the *j* symbol is missing!)
Putting this in (4) yields

$$Ni = \Phi\left(\frac{R_{AIR}}{\mu_p'} + j\frac{R_{AIR}}{\mu_p''}\right) \qquad (11)$$

Thus, relative to the flux Φ, the applied mmf appears to have two components as shown in figure 3.

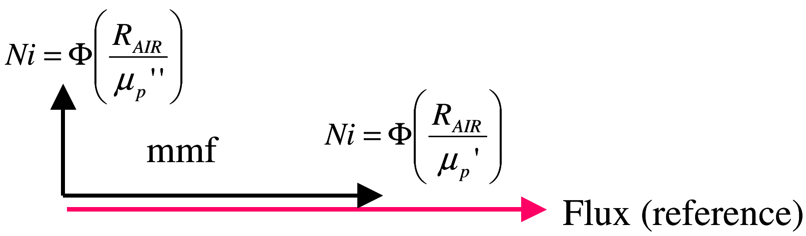

Figure 3. The two components of mmf

If we now differentiate the flux vector (multiply by $j\omega$) to get a reference *voltage* vector we get the following diagram, figure 4.

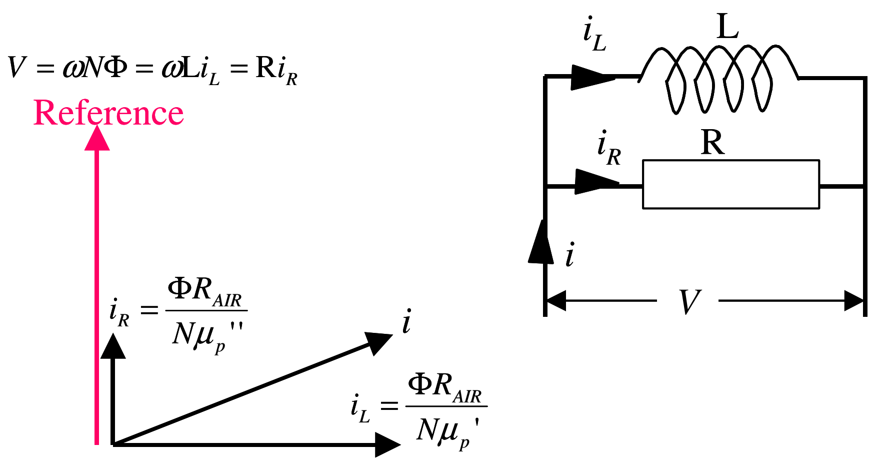

Figure 4. The two components of current

The in-phase current i_R obtained from the *j* component of (11) is that current which would flow through a parallel resistance $R = \frac{\omega N^2 \mu_p''}{R_{AIR}}$, while the quadrature component i_L is the current that would flow through the parallel inductance $L = \frac{N^2 \mu_p'}{R_{AIR}}$.

2. Measurement Considerations

When we perform a measurement we will obtain a phase angle α between voltage and current, figure 5.

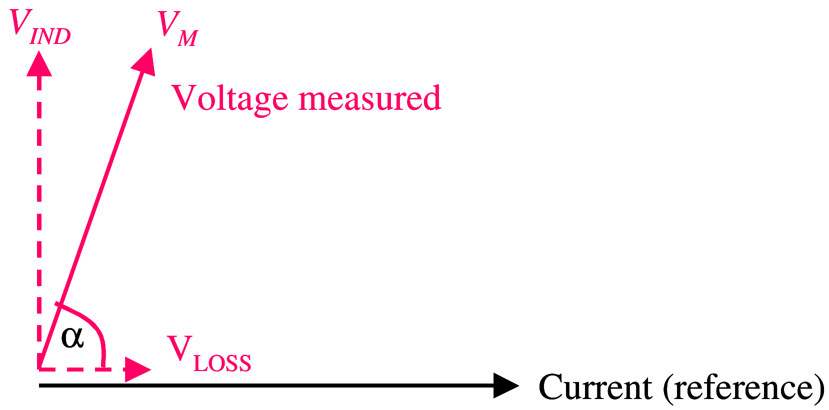

Figure 5. Measured data

If there is no air gap in the core, we can derive the series values of complex permeability directly. We need to derive the in-phase and quadrature components from the measured voltage V_M. Using $V_{LOSS} = V_M \cos\alpha$ for the in-phase component and equating this with (8) we obtain

$$\mu_s'' = \frac{V_M R_{AIR} \cos\alpha}{\omega N^2 i} \qquad (12)$$

Similarly using $V_{IND} = V_M \sin\alpha$ for the quadrature component and equating with (9) we get

$$\mu_s' = \frac{V_M R_{AIR} \sin\alpha}{\omega N^2 i} \qquad (13)$$

3. Correction for an air gap.

If the magnetic circuit has an air gap, then (1) must be replaced by

$$\Phi = \frac{Ni}{R + R_{GAP}} \qquad (14)$$

where the gap reluctance is obtained from the gap width g as

$$R_{GAP} = \frac{g}{\mu_0 A} \qquad (15)$$

Equation (4) becomes

$$\Phi = \frac{Ni}{\frac{R_{AIR}}{\mu_R} + R_{GAP}} \qquad (16)$$

This is best solved using (10), the parallel representation for μ_R, giving

$$\Phi = \frac{Ni}{\frac{R_{AIR}}{\mu_p'} + j\frac{R_{AIR}}{\mu_p''} + R_{GAP}} \qquad (17)$$

which can be manipulated into

$$Ni = \Phi\left(\left(\frac{R_{AIR}}{\mu_p'} + R_{GAP}\right) + j\frac{R_{AIR}}{\mu_p''}\right) \qquad (18)$$

This yields the current component that is in-phase with the *voltage* as $i_{\mathrm{R}} = \dfrac{\Phi R_{AIR}}{N\mu_p''}$

Equating this with the in-phase current $i_{\mathrm{R}} = i_M \cos\alpha$ as measured, and using $\Phi = \dfrac{V_M}{\omega N}$ we get the wanted value of

$$\mu_p'' = \frac{V_M R_{AIR}}{\omega N^2 i_M \cos\alpha} \qquad (19)$$

Similarly equating the quadrature component $i_{\mathrm{L}} = \dfrac{\Phi}{N}\left(\dfrac{R_{AIR}}{\mu_p'} + R_{GAP}\right)$ with the measured current $i_{\mathrm{L}} = i_M \sin\alpha$ yields

$$\mu_p' = \frac{V_M R_{AIR}}{\omega N^2 i_M \sin\alpha - V_M R_{GAP}}\,. \qquad (20)$$

Figure 6 shows the measured current components against the measured voltage.

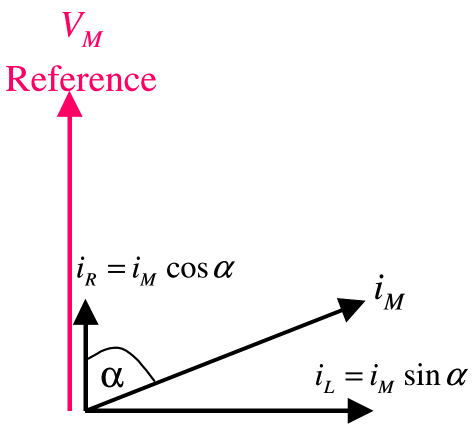

Figure 6. The two current components

[Note that by putting R_{GAP}=0, (19) and (20) yield the parallel complex permeability from measurements on a core with no air gap.]

To convert these measurements into the series equivalents we can use the relationships

$$\mu_s' = \frac{\mu_p'}{1+\tan^2\delta} \text{ and } \mu_s'' = \frac{\mu_p''\tan^2\delta}{1+\tan^2\delta}$$

but first we have to calculate the value of $\tan\delta$ that applies to only the core material [we must not use the loss angle δ=(90°-α) as measured with the air gap present] . This is given by the ratio of (20) to (19) as

$$\tan\delta = \frac{\mu_p'}{\mu_p''} = \frac{\omega N^2 i_M \cos\alpha}{\omega N^2 i_M \sin\alpha - V_M R_{GAP}} \qquad (21)$$