On Charge Movement through a Magnetic Vector Potential Field

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1. Introduction

This paper studies charge moving in an **A** field from the energy perspective. A charge q placed in an **A** field is said to have a *canonical momentum* q**A**, but what does this mean? *Canonical* means "*by definition*", so is q**A** simply a definition without meaning, certainly the charge q does not appear to possess mechanical momentum of that value? And since the product q**A** really does have the dimensions of momentum (mass × velocity), what happens if q moves along the **A** field with velocity **v**? The product of momentum×velocity is energy, does qAv signify an energy and if so where does it manifest itself? Also, since power is energy rate, $qA \cdot dv/dt$ signifies a power transfer associated with the charge acceleration, but where does that transfer take place? In this paper we show that these energy considerations apply both to the *source* of the **A** field and to the force that is moving the charge. The moving charge has potential energy qAv which can be accessed via an induced force on the charge while it moves out of the **A** field region. If the charge gains that energy by being accelerated from rest up to velocity v while within the **A** field, the energy is taken from the current generator that supplies the **A** field . Conversely, if the charge loses that energy by being decelerated while within the **A** field, the energy is given back to the current generator.

2. A current loop as an A field source

Consider an arbitrary closed current loop driven by a constant current generator I. Let this loop create an **A** field at some arbitrary position, as shown in Figure 1.

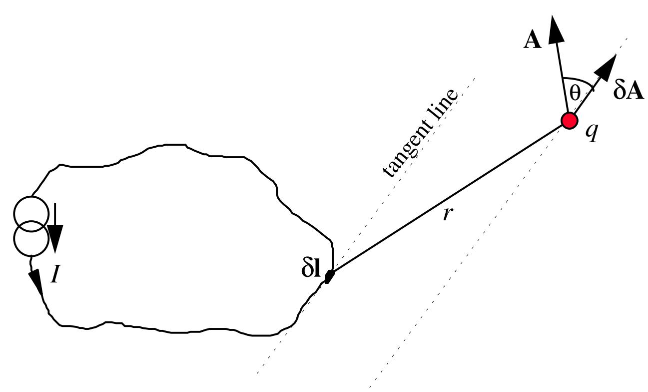


Figure 1.

Take a small element of length **δl** creating the differential field component **δA**. **δA** lies parallel to **δl** and is given by

$$\delta\mathbf{A} = \frac{\mu_0 I \delta\mathbf{l}}{4\pi r} \qquad (1)$$

where r is the distance from the current element to the field point. Note that **δA** does not necessarily lie along **A**, let the angle between **δA** and **A** be θ. Thus **δA** contributes a value

$\delta A\cos\theta$ to the total field **A**, and we can express the **A** field magnitude at this point by the integral

$$A = |\mathbf{A}| = \frac{\mu_0 I}{4\pi}\oint \frac{\cos\theta}{r}\cdot dl \qquad (2).$$

where θ and r are variables of the integration. We cannot use this integral to evaluate A since it assumes knowledge of the vector direction of **A** to obtain θ, so what use does it have? In the next section we derive another equation that has the identical integral, then we can eliminate this integral without the need for a solution.

Now place a charge q at the field point and let it be accelerated along the **A** field direction, see Figure 2.

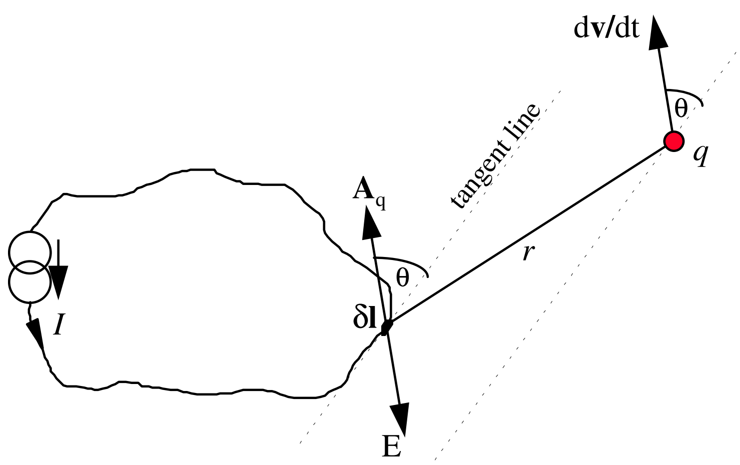


Figure 2

We wish to evaluate the voltage induced into the current loop, and we start by finding the voltage induced into the element $\delta\mathbf{l}$. We find that the moving charge produces an $\mathbf{A}_q$ field at $\delta\mathbf{l}$ related to its velocity **v** as

$$\mathbf{A}_q = \frac{\mu_0 q\mathbf{v}}{4\pi r} \qquad (3)$$

and since **v** is changing with time we get an **E** field from

$$\mathbf{E} = -\frac{d\mathbf{A}_q}{dt} = -\frac{\mu_0 q}{4\pi r}\frac{d\mathbf{v}}{dt} \qquad (4)$$

This **E** field induces a differential voltage δV along δl

$$\delta V = -\frac{\mu_0 q\delta l}{4\pi}\frac{dv}{dt}\frac{\cos\theta}{r} \qquad (5)$$

giving the total voltage induced into the loop as

$$V = -\frac{\mu_0 q}{4\pi}\frac{dv}{dt}\oint\frac{\cos\theta}{r}\cdot dl \qquad (6)$$

where both θ and r are variables. The integral part of this equation is seen to be identical to that in (2), so combining (6) with (2) eliminates the integral and yields

$$V = -\frac{qA}{I}\frac{dv}{dt} \tag{7}$$

This voltage appears across the current generator, indicating a power flow *P* which we can express by rearranging (7) as

$$P = VI = -qA\frac{dv}{dt} \tag{8}$$

Thus acceleration of the charge q along the **A** *field direction has caused a power flow at the energy source which created the* **A** *field. This power flow can be negative or positive depending on the direction of the acceleration, with or against the* **A** *field, and the polarity of the charge q.* For a charge that is initially at rest and is suddenly accelerated to a velocity **v** over a small region of space where **A** is constant, the energy found by integrating the power impulse (8) over the impulse time has a magnitude qAv. There is no reaction on the force creating the acceleration, so where does this gain or loss of energy then appear? In the next section we show that the energy is accessible when the charge moves out of the **A** field region.

3. Longitudinal Induction

Phipps [1] argues that the equation $\mathbf{E} = -\frac{d\mathbf{A}}{dt}$ is normally applied to a field point fixed in space where the vector potential **A** changes with time, where one should really use the partial derivative $\frac{\partial \mathbf{A}}{\partial t}$. When considering a moving charge it is necessary to use the *total* time derivative $\frac{d\mathbf{A}}{dt}$ (Phipps uses $\frac{D\mathbf{A}}{Dt}$ to avoid confusion with the usual notation for the derivative of a function of a single variable) which takes account not only of the time variation of **A** (for a fixed point in space) but also the spatial variation in **A** (for a moving point). This then introduces a convection term associated with charge movement through a non-uniform **A** field

$$\mathbf{E} = -(\mathbf{v}\cdot\nabla)\mathbf{A} \tag{9}$$

where $(\mathbf{v}\cdot\nabla)$ is a particular scalar convection operator (see Appendix B). *Note, unlike classical* $\mathbf{E} = \mathbf{v}\times\mathbf{B}$ *induction this* ***E*** *field has a component that lies along the velocity direction.*

Wesley [3] points out that equation (9) may be rewritten using a vector identity to give

$$\mathbf{E} = \mathbf{v}\times(\nabla\times\mathbf{A}) - \nabla(\mathbf{v}\cdot\mathbf{A}) \tag{10}$$

which becomes

$$\mathbf{E} = \mathbf{v}\times\mathbf{B} - \nabla(\mathbf{v}\cdot\mathbf{A}) \tag{11}$$

Thus (9) which expresses the **E** field as "seen" by the moving charge already contains the classical $\mathbf{E} = \mathbf{v}\times\mathbf{B}$ induction, as it should. We can now include this in the general equation for electric fields as

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t} - \nabla(\mathbf{v}\cdot\mathbf{A}) + \mathbf{v}\times\mathbf{B} \tag{12}$$

The $-\nabla(\mathbf{v}\cdot\mathbf{A})$ *term is not mentioned in classical texts on electromagnetism, this neglected term could lead to new discoveries in this field.* It may be noted that the scalar product $\mathbf{v}\cdot\mathbf{A}$ yields a scalar potential field as "seen" by the moving charge, which must be added to the scalar potential ϕ from any spatial charge distribution. Then (12) can be rewritten

$$\mathbf{E} = -\nabla(\phi + \mathbf{v}\cdot\mathbf{A}) - \frac{\partial \mathbf{A}}{\partial t} + \mathbf{v}\times\mathbf{B} \tag{13}$$

Clearly the scalar potential $\mathbf{v}\cdot\mathbf{A}$ is maximum when the charge travels along the **A** direction, and for uniform **A** field is a constant. When **A** is non-uniform then the charge will "see" a potential gradient *along its trajectory*, i.e. along the velocity direction. If we choose a Cartesian co-ordinate system where the velocity lies along the x co-ordinate, the charge will endure a force

$$F_x = q\frac{\partial(\mathbf{v}\cdot\mathbf{A})}{\partial x}. \tag{14}$$

Phipps quotes the Marinov motor [2] as evidence that this is a real effect, this motor has also been considered by Wesley [3]. Unfortunately, due to the low drift velocity of conduction electrons (**v** in equation (14)) the forces exerted on them are so small that the measured torque on the Marinov motor has been either (a) wrongly attributed to flux leakage and the classical $\mathbf{v}\times\mathbf{B}$ Lorentz force, (b) considered to be a measurement artifact or (c) considered to be of no real interest.

Like any electric motor the Marinov version can be mechanically driven to perform as a generator. This has the advantage that the driven velocity **v** in (14) can be significantly greater than the electron drift velocity that applies to the motor version, yielding readily detectable voltage that cannot be construed as a measurement artifact. Surprisingly it appears that this simple generator experiment to validate (14) has not been carried out, the nearest to be found is the Distinti Paradox 2 [4] which is an overly complex version of the Marinov motor. To correct this omission the author, with the help of a UK slip-ring manufacturer (BGB Engineering Ltd.), has carried out his own tests that are reported here for the first time (see Appendix A for the configuration). Using a 100mm diameter slip-ring rotating at speeds up to 1000 RPM, voltages up to 3.3mV DC were recorded from brushes at diametrically opposite positions. Although this experiment used open magnets that supplied a **B** field to the moving ring, the geometry was such that the $\mathbf{v}\times\mathbf{B}$ Lorentz force could not account for the induced voltage, whereas integration of (9) along the conducting surface between the brushes yielded a voltage in agreement with the measurement (see Appendix B for the math). In view of the importance of equations (9) to (14) to the conclusions reached in this paper, the slip-ring experiment should be performed by other research establishments.

4. Energy conservation

Now with the use of (9) it is possible to answer the question posed in section 1. If we consider within an **A** field a charge that is initially stationary, but is then suddenly accelerated along the **A** direction to a velocity v over a small time period Δt, the mechanical energy needed to do this is the kinetic energy $0.5mv^2$. If the charge is now allowed to escape from the **A** field region to where A=0, the electric field from (9) will apply force to change the velocity to a new value v_0. The final kinetic energy $0.5mv_0^2$ differs from that supplied and it is found by integrating (9) over the escape path that the difference is exactly qAv. That gain or loss of energy is fully accounted for by integrating equation (8) over the acceleration impulse time Δt, yielding the initial energy taken from or given back to the current source. Thus we have a full accounting for the energies involved. We can obtain greater KE of a moving charge than mechanical energy supplied, that increase in KE taking place as the potential energy qAv (initially taken from the current source creating the **A** field) is converted to KE by force from the electric field (9) as the charge moves out of the **A** field. Total energy is conserved.

Figure 3 shows an electron passing a current loop but here the velocities are assumed to be held constant over the approach and escape phases. At the point of nearest approach the electron is suddenly accelerated from its initial low approach velocity v_1 to a high recession velocity v_2. At that acceleration point the A field value is A_{max}. During the approach phase equation (14) produces the force F_1 opposing the movement, hence energy must be supplied as force on the moving charge to maintain v_1, and the integration of F_1 from large distance up to the near point yields such an input energy of value $eA_{max}v_1$. Similarly equation (14) produces a force F_2 aiding the escape velocity v_2, where integration of F_2 yields an output energy $eA_{max}v_2$. Over the acceleration phase the current generator supplies an energy pulse of value $eA_{max}(v_2-v_1)$.

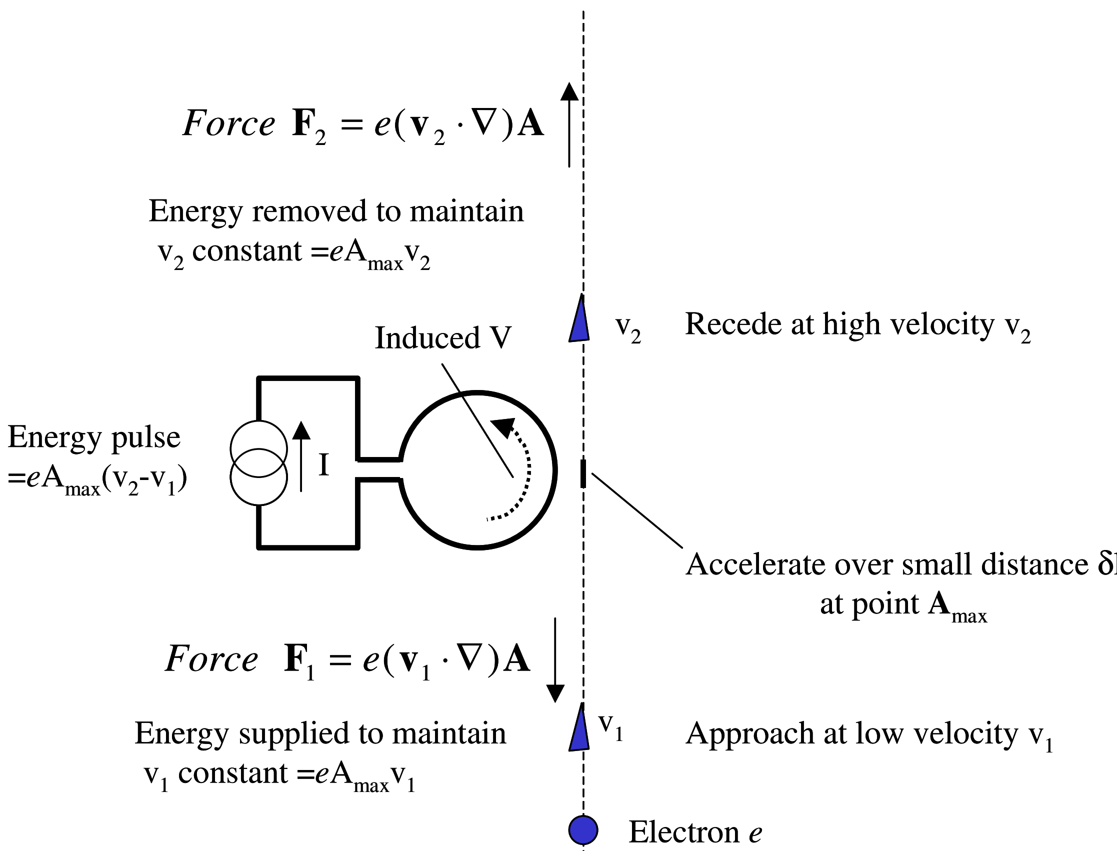


Figure 3. Electron Passing a Current loop

Thus the energy audit reveals

Input Energy			**Output Energy**	
Initial KE	$0.5mv_1^2$		F_2 integral	$eA_{max}v_2$
F_1 integral	$eA_{max}v_1$		Final KE	$0.5mv_2^2$
Acceleration energy	$0.5m(v_2^2-v_1^2)$			
From current supply	$eA_{max}(v_2-v_1)$			

Output = Input, energy is conserved.

5. Using a Permanent Magnet

It will be noted that the current supplying the **A** field is DC, and that (6) represents a unidirectional voltage impulse which "loads" the current source. If we replace the current loop with a permanent magnet, do we now have the means for extracting energy from it? If we replaced the single current loop with an array of current loops, the electric field radiated by the accelerating charge will induce voltage into each of them, so it is possible that the same argument will apply if the loops have atomic dimensions. In the energy audit above, the acceleration energy $0.5m(v_2^2-v_1^2)$ taken from the current source is now supplied by the magnet, resulting in greater output than input. There are interesting times ahead as this possibility gets explored.

6. Historical Note

Sommerfeld [5] points out that in 1903 Schwarzschild introduced his "electrokinetic potential" $L = (\phi - \mathbf{v} \cdot \mathbf{A})$, so it is over 100 years since the scalar product $\mathbf{v} \cdot \mathbf{A}$ was recognized as a potential. A Google search on "electrokinetic potential" plus Schwarzschild confirms that. Schwarzschild's electrokinetic potential L is really a *potential difference* which when multiplied by the charge density forms a relavistic invariant, which was important in the development of his Principle of Least Action. The mathematical intricacies of that development is of little interest to engineers, they are interested in electric and magnetic forces that can do work. The **E** field (13) is just that, hence it would be more sensible if the electrokinetic potential capable of doing work in a fixed laboratory frame were the *sum* $(\phi + \mathbf{v} \,.\, \mathbf{A})$**,** not the difference. Thus a charge q moving at velocity **v** through an **A** field can be considered to have a *kinetic* potential $\mathbf{v.A}$, or a kinetically derived potential energy $q(\mathbf{v.A})$. Note this is a maximum when **v** is parallel to **A**, when the charge travels along the **A** field, and has a value qAv . This is in exact agreement with the energy available from the motion derived **E** field (9) and with the energy extracted from the source of the **A** field (8).

References.

[1] Thomas E. Phipps, Jr., “Invariant Electromagnetism: Necessity and Sufficiency”

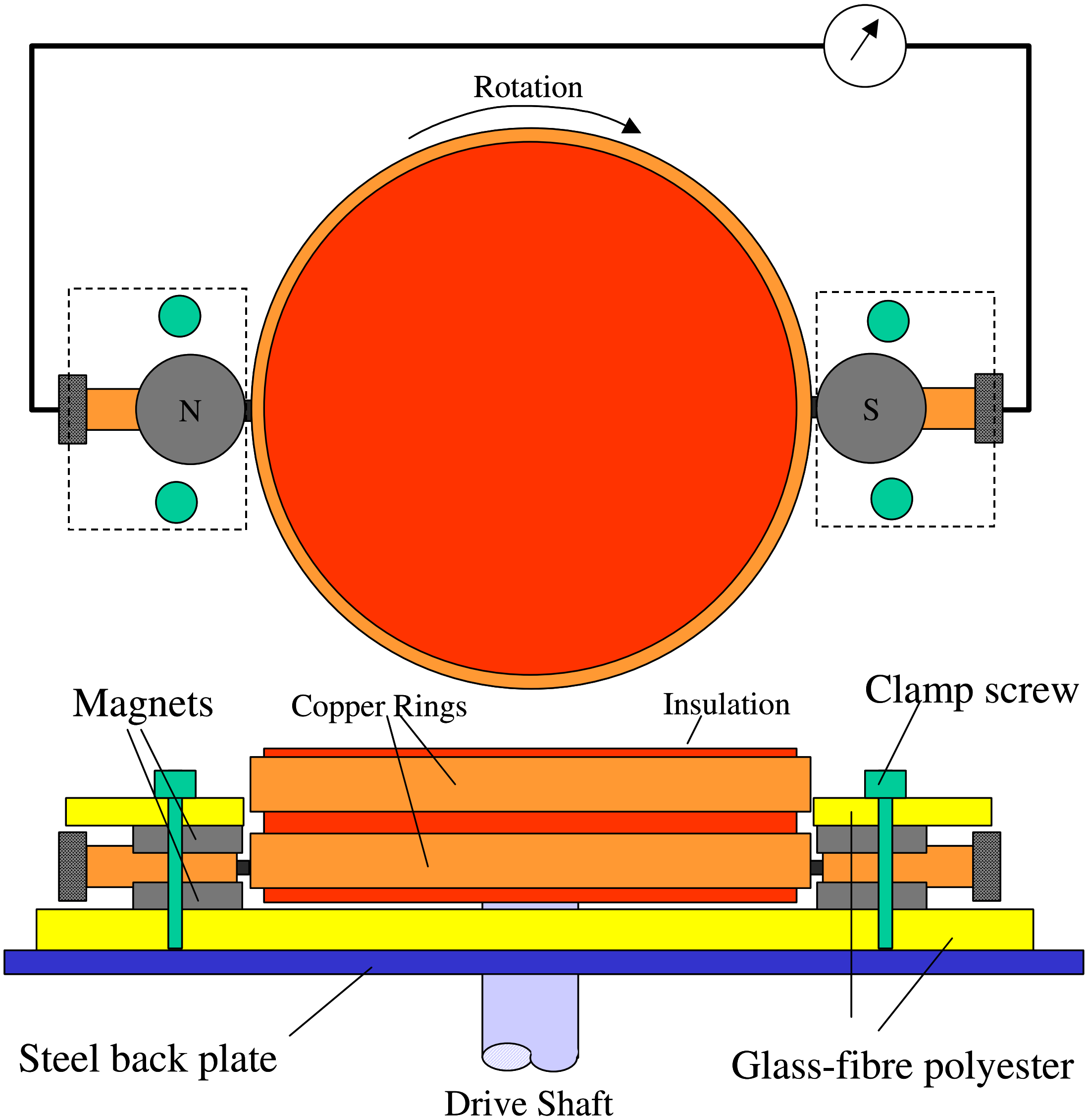
[2] Thomas E. Phipps, Jr., “Observations of the Marinov Motor”, APEIRON Vol. 5 Nr.3-4, July-October 1998.

[3] J. P Wesley, “The Marinov Motor, Notional Induction without a Magnetic B Field”**,** APEIRON Vol. 5 Nr.3-4, July-October 1998.

[4] Distinti Paradox 2 http://www.newelectromagnetism.com/

[5] Arnold Sommerfeld:- 1948 "Lectures on Theoretical Physics" vol. 3 "Electrodynamics" (translated by Edward G. Ramberg and published by Academic Press in 1964).

Appendix A. Marinov Generator



Marinov Generator

Appendix B. Vector evaluation.

In cylindrical coordinates the vector $(\mathbf{v}\cdot\nabla)\mathbf{A}$ is given by

$$(\mathbf{v}\cdot\nabla)\mathbf{A} = \mathbf{a}_r\left[v_r\frac{\partial A_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial A_r}{\partial \theta} - \frac{v_\theta A_\theta}{r} + v_z\frac{\partial A_r}{\partial z}\right]$$
$$+\mathbf{a}_\theta\left[v_r\frac{\partial A_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{v_\theta A_r}{r} + v_z\frac{\partial A_\theta}{\partial z}\right] \tag{B1}$$
$$+\mathbf{a}_z\left[v_r\frac{\partial A_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial A_z}{\partial \theta} + v_z\frac{\partial A_z}{\partial z}\right]$$

where $\mathbf{a}_r$, $\mathbf{a}_\theta$ and $\mathbf{a}_z$ are the unit vectors. In the experiment we have a cylindrical surface rotating about the z axis where v_r, v_z and A_z are everwhere zero, hence this reduces to

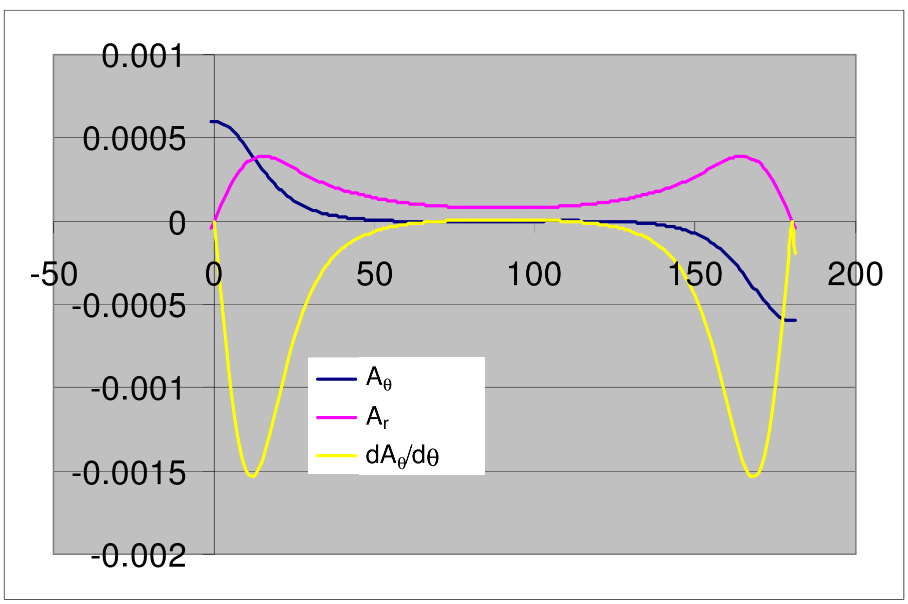
$$(\mathbf{v}\cdot\nabla)\mathbf{A} = \mathbf{a}_r\left[\frac{v_\theta}{R}\frac{\partial A_r}{\partial \theta} - \frac{v_\theta A_\theta}{R}\right] + \mathbf{a}_\theta\left[\frac{v_\theta}{R}\frac{\partial A_\theta}{\partial \theta} + \frac{v_\theta A_r}{R}\right] \tag{B2}$$

where we have replaced the variable r with the fixed radius R of the slip-ring.

Only the θ component can induce voltage along the conductor, hence the desired integral to obtain that voltage is

$$V = \int_0^\pi E_\theta R.d\theta = v_\theta\int_0^\pi\left(\frac{\partial A_\theta}{\partial \theta} + A_r\right).d\theta \tag{B3}$$

Typical plots of A_θ, A_r and $\partial A_\theta/\partial\theta$ plotted against angle θ are shown in the figure.



It is seen that the negative area under $\partial A_\theta/\partial\theta$ exceeds the positive area under A_r hence the integral (B3) yields a finite result.