Complex Permeability Modeling

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1. Introduction

In their paper [1] Caltun et al give formulae for permeability taken from Nakamura et al [2]. According to Caltun, Nakamura decomposed the susceptibility spectra into a spin rotational component $\chi_{sp}(\omega)$ and a domain wall component $\chi_{dw}(\omega)$, leading to the equation for permeability as

$$\mu(\omega) = 1 + \chi_{sp}(\omega) + \chi_{dw}(\omega) \quad . \tag{1}$$

These components were then given as

$$\chi_{sp}(\omega) = \frac{K_{sp}}{1 = j\left(\dfrac{\omega}{\omega_{sp}}\right)} \tag{2}$$

$$\chi_{dw}(\omega) = \frac{K_{dw}\omega_{dw}{}^{2}}{(\omega_{dw}{}^{2} - \omega^{2} + j\beta\omega)} \tag{3}$$

where K_{sp} is the static (or low frequency) susceptibility due to spin rotation, ω_{sp} the spin relaxation frequency, K_{dw} the static susceptibility of the domain wall motion, ω_{dw} the domain wall resonance and β the damping factor.

2. Analysis

Equations (2) and (3) can be separated into real χ' and imaginary χ'' components

$$\chi_{sp}' = \frac{K_{sp}}{1 + \left(\dfrac{\omega}{\omega_{sp}}\right)^{2}} \tag{4}$$

$$\chi_{sp}'' = -\frac{K_{sp}\left(\dfrac{\omega}{\omega_{sp}}\right)}{1 + \left(\dfrac{\omega}{\omega_{sp}}\right)^{2}} \tag{5}$$

$$\chi_{dw}' = \frac{K_{dw}\omega_{dw}{}^{2}(\omega_{dw}{}^{2} - \omega^{2})}{(\omega_{dw}{}^{2} - \omega^{2})^{2} + (\beta\omega)^{2}} \tag{6}$$

$$\chi_{dw}'' = -\left(\frac{K_{dw}\omega_{dw}{}^{2}\beta\omega}{(\omega_{dw}{}^{2} - \omega^{2})^{2} + (\beta\omega)^{2}}\right) \tag{7}$$

Since the relative permeability μ_R is related to magnetic susceptibility χ by $\mu_R = 1 + \chi$ we can use the real and imaginary components of χ to obtain the series form of complex permeability

$$\mu_R = \mu' - j\mu'' \tag{8}$$

where

$$\mu' = 1 + \chi_{sp}' + \chi_{dw}' \tag{9}$$

and

$$\mu'' = \chi_{sp}'' + \chi_{dw}'' \tag{10}$$

3. Spin Rotation Component.

When we analyse the spin rotation susceptibilities we find that for frequencies much lower than ω_{sp} the imaginary component is negligible while the real component has a value of K_{sp}. The real component falls to a value $K_{sp}/2$ at the cut-off frequency ω_{sp} then continues falling at a rate of 12dB per octave in the manner of a low pass filter characteristic. The imaginary component rises from zero to a peak value of $K_{sp}/2$ at the frequency ω_{sp} then falls at a rate of 6dB per octave thereafter. These characteristics are shown in Figure 1 for K_{sp} = 200 and f_{sp} = 9MHz (*note this does not represent any particular ferrite*). Two chart versions are shown, one with a logarithmic vertical axis and the other with linear scaling.

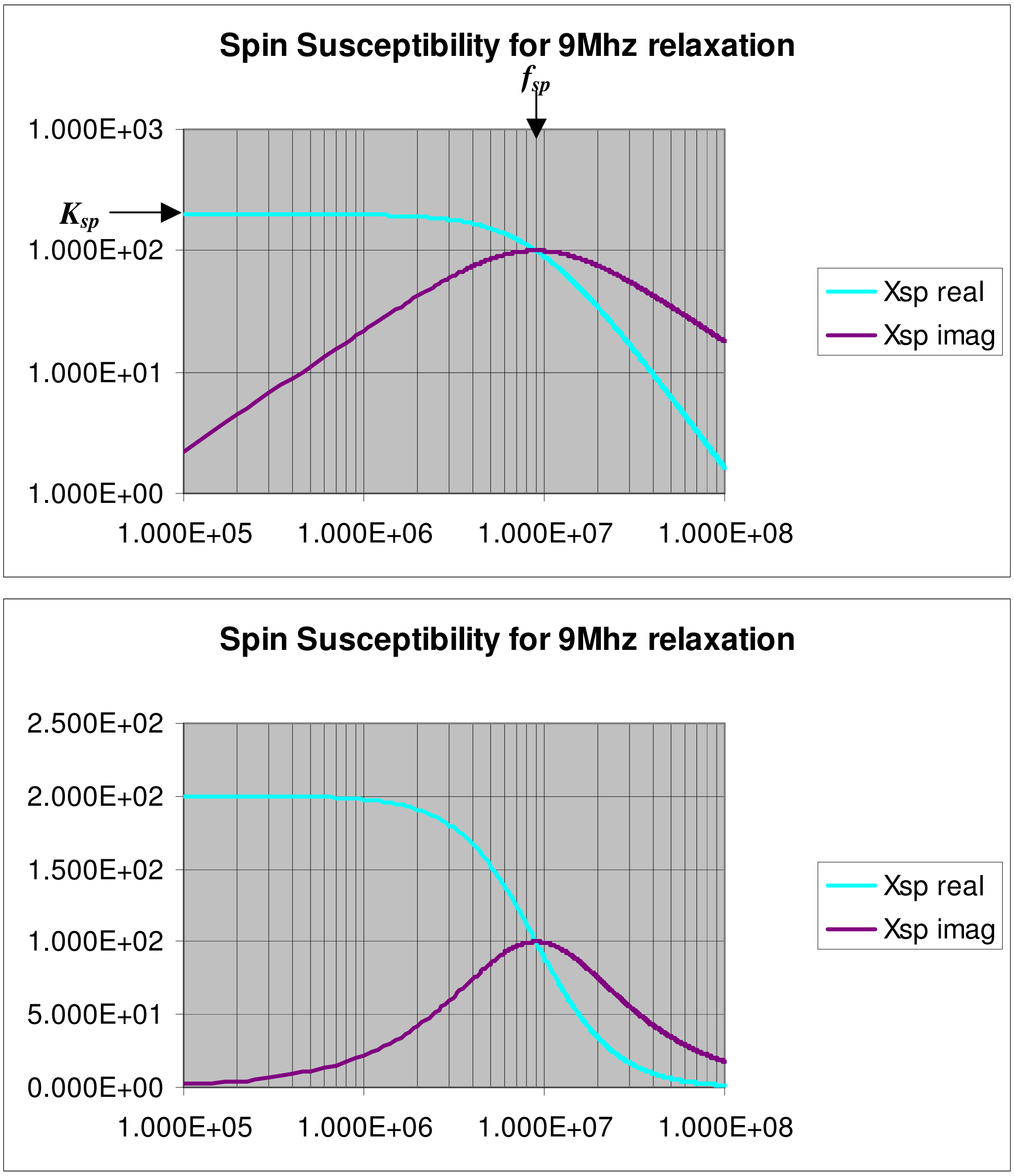

Figure 1. Spin Components of Susceptibility

4. Domain Wall Components of Susceptibility

Turning now to the domain wall contribution, at low frequencies the real component has a value of K_{dw} while the imaginary component is effectively zero. The real component then rises to a peak value at a frequency

$$\omega_1 = \sqrt{\omega_{dw}^{\;2} - \beta\omega_{dw}}\;, \tag{11}$$

passes through zero at frequency ω_{dw} to reach a maximum negative value at a frequency of

$$\omega_2 = \sqrt{\omega_{dw}^{\;2} + \beta\omega_{dw}}\;. \tag{12}$$

At higher frequencies the real component decays back to zero. This S shaped curve is a characteristic of resonance. The peak values of this curve are given by

$$\max \chi_{dw}' = K_{dw}\frac{\omega_{dw}^{\;2}}{\beta(2\omega_{dw} - \beta)} \tag{13}$$

$$\min \chi_{dw}' = -K_{dw}\frac{\omega_{dw}^{\;2}}{\beta(2\omega_{dw} + \beta)} \tag{14}$$

The imaginary component rises from zero to a peak value at the resonant frequency ω_{dw}. This peak value is given by

$$\text{peak}\chi_{dw}'' = K_{dw}\frac{\omega_{dw}}{B} \tag{15}$$

These features are illustrated in Figure 2 for $K_{dw} = 100$, f_{dw}=3MHz and β=1MHz.

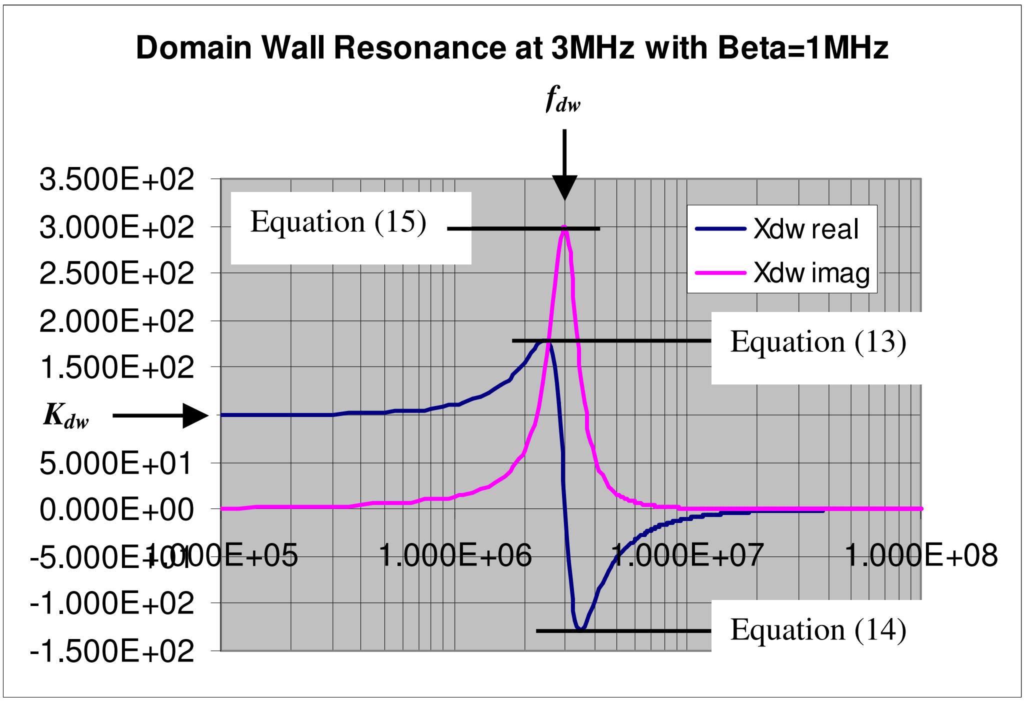

Figure 2. Domain Wall components of Susceptibility

When the susceptibilities shown in figures 1 and 2 are combined into the complex permeability (8) via (9) and (10) the resulting μ' and μ'' values are shown in Figure 3.

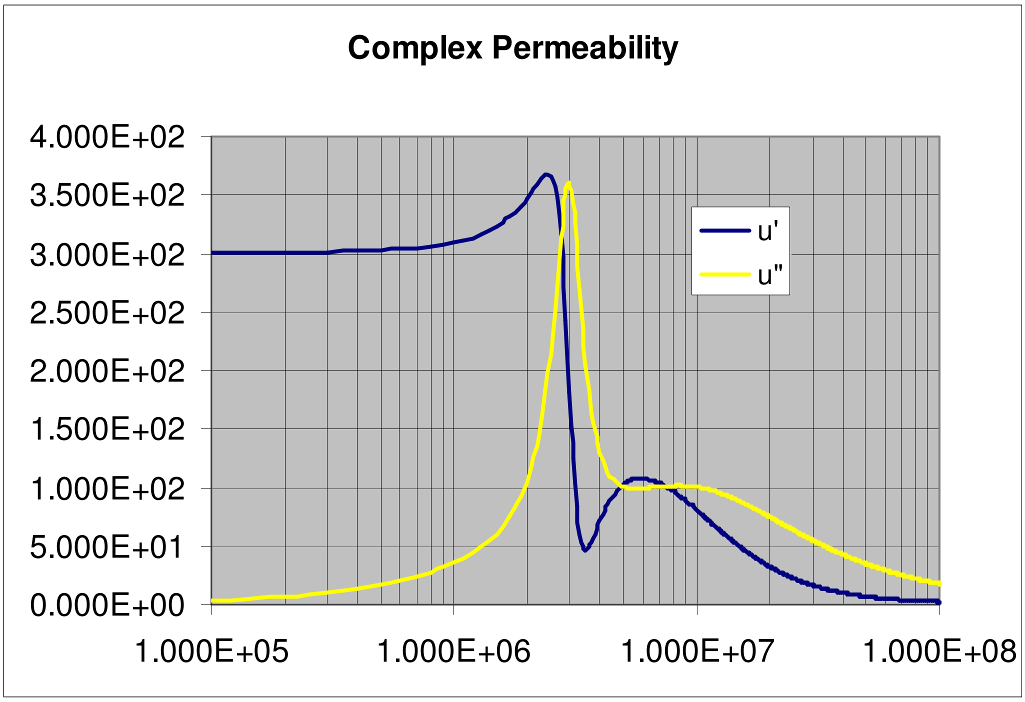

Figure 3. Complex Permeability

5. Discussion

Any known complex permeability spectrum can be curve fitted to the Nakamura equations, thus giving an indication of the underlying separate spin rotation relaxation frequency and domain wall resonance features. Other papers relate these to production characteristics of the ferrite such as chemical composition and grain size, hence this should assist in the design of a ferrite better suited to the intended operation.

References

[1]. O.F Caltun, L. Spinu, Al. Stancu, L.D. Thung, W. Zhou, “Study of the microstructure and of the permeability spectra of Ni-Zn-Cu ferrites”. Journal of Magnetism and Magnetic Materials 242-245 (2002) 160-162

[2]. T. Nakamura, J. Appl. Phys. 88 (2000) 348