

# AN IMPROVED METHOD FOR VIRTUAL AIR GAP LENGTH COMPUTATION

A. Konrad<sup>1</sup>, J. F. Brudny<sup>2</sup>

(1) Dept. of E.C.E., Univ. of Toronto, 10 King's College Rd., Toronto, ON, M5S 3G4, Canada

(2) Elec. Eng. Dept., LSEE, University of Artois, Technoparc Futura, Bethune, 62400 France

## Introduction

An approach to the analysis of a single-phase transformer with a virtual air gap is presented. The transformer geometry is shown in Fig.1. The transformer has 4 auxiliary windings embedded in its core and connected to a dc supply. The purpose of these windings is to locally saturate the core. By changing the magnetic permeability of the ferromagnetic core material in the region of the auxiliary windings, a virtual air gap is created. An equivalent mechanical air gap length can be associated with the virtual air gap. It results from an increase in core reluctance with local saturation and is obtained by an analytical approach based on finding the distribution of the magnetic field in the core produced by the auxiliary windings. The accurate calculation of the magnetic field is necessary to obtain both the operating point of the core and the change in core reluctance. The proposed method consists of 5 steps. Steps 1, 2 and 5 are as described in [1]. Steps 3 and 4 are new [2]. The result obtained from the proposed method is compared with experimental and analytical results presented in [1].

## Description and Application of the Method

**Step 1** is to obtain the reluctance of the core with uniform cross section *without* local saturation. The ac voltage applied to the main windings produces a magnetic flux  $\Phi_m$  in the core. The corresponding current is obtained from the magnetization curve and the magnetomotive force (mmf) required without local saturation is found. Denoting the mmf by  $\mathfrak{Z}_m$ , the reluctance becomes  $\mathfrak{R}_m = \mathfrak{Z}_m / \Phi_m = 23.93 \text{ A-t} / 5.359 \times 10^{-4} \text{ Wb} = 44653.85 \text{ H}^{-1}$ .

**Step 2** is the calculation of the magnetic field  $H_a$  when a dc current of  $I_a = \pm 10 \text{ A}$  is applied to the auxiliary windings. The region of core across the center of the auxiliary windings is divided into  $m$  tubes of different lengths (Fig.2). An average value of  $H_a$  is obtained by the image method in [1].  $H_a$  is used to obtain the flux  $\Phi_a$  around the auxiliary windings.

**Step 3** is to take into account the induction in the auxiliary windings.  $\Phi_m$  induces a current in the auxiliary windings, which sets up a magnetic flux that opposes  $\Phi_m$ . However, instead of a decrease in main flux, an equivalent effect, the reduction of  $H_a$  to  $H_{ia}$  is considered.  $H_{ia}$  along the mean core length is calculated using an equivalent reduced auxiliary winding current  $I_{ia}$ . Fig.3 compares the distributions of  $H_a$  and  $H_{ia}$  in one quarter of the core.

**Step 4** is the calculation of the equivalent  $\Phi$ - $\mathfrak{Z}$  curve of the core [3] by division of the core into  $n$  slices according to the variation of  $H_{ia}$  along the mean path length of the core. From the equivalent  $\Phi$ - $\mathfrak{Z}$  curve the total mmf,  $\mathfrak{Z}$ , is obtained. Due to symmetry, only one quarter

of the core is considered. In this example  $n = 68$  (Fig.4). Each slice has a specific value of  $H_{ia}$  and its own magnetization curve reformulated in terms of  $\Phi$  and  $\mathfrak{Z}$  (Fig.5). The equivalent  $\Phi$ - $\mathfrak{Z}$  curve is obtained from the sum of the mmf-s of each slice for the same value of  $\Phi_m$  [3]. The total mmf required to establish  $\Phi_m$  with local saturation is obtained from the equivalent  $\Phi$ - $\mathfrak{Z}$  curve (Fig.6). For a magnetic flux of  $\Phi_m = 5.359 \times 10^{-4} \text{ Wb}$  with local saturation corresponding to  $I_a = \pm 10 \text{ A}$ , the total mmf required is  $\mathfrak{Z} = 239.43 \text{ A-t}$ .

**Step 5** consists of obtaining the reluctance of the core *with* local saturation:  $\mathfrak{R} = \mathfrak{Z} / \Phi_m = 239.43 \text{ A-t} / 5.359 \times 10^{-4} \text{ Wb} = 446781.12 \text{ H}^{-1}$ . The increase in core reluctance is  $\Delta \mathfrak{R}_m = \mathfrak{R} - \mathfrak{R}_m = 402127.26 \text{ H}^{-1}$ . Finally, the equivalent length of the virtual air gap is obtained as  $\ell_g = \Delta \mathfrak{R}_m \mu_0 S = 2.5 \text{ mm}$  where  $S = (0.066 \text{ m}) \times (0.075 \text{ m}) = 0.00495 \text{ m}^2$  is the core cross-sectional area. For comparison, the computed and experimental values of  $\ell_g$  given in [1] for the same level of saturation are 4.5mm and 2.59mm, respectively.

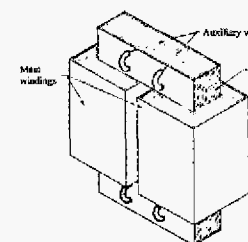


Fig. 1 - Transformer geometry showing 3D windings and core.

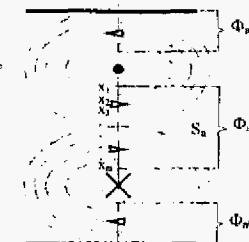


Fig. 2 - Tubes  $x_1$  through  $x_m$  around an auxiliary winding.

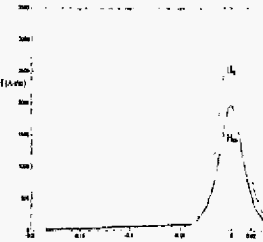


Fig. 3 - 1D fields  $H_a$  and  $H_{ia}$  along mean length of  $1/4$  core.

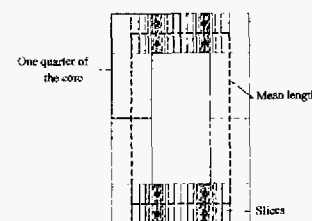


Fig. 4 - Subdivision into  $n$  slices according to the variation of  $H_{ia}$ .

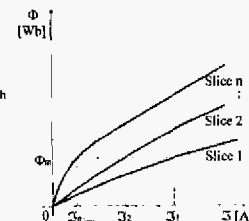


Fig. 5 - Typical magnetization curves of the  $n$  core slices.

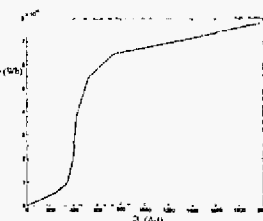


Fig. 6 - Equivalent  $\Phi$ - $\mathfrak{Z}$  curve corresponding to  $I_a = \pm 10 \text{ A}$ .

## References

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