Magnetic Delay Lines

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This document discusses the theory behind a magnetic delay line where a series of coils are wound on a ferrous rod, each coil "shorted" by a capacitor.

Comparison with Twin Wire Transmission Line

We actually need two ferrous rods to act as a balanced delay line. When such a system is transporting energy there is a magnetic potential difference (mmf) between the two lines, so an H field can be found there. Normally such an H field is considered to be leakage, but in fact it exists as part of the Poynting vector transporting the energy, so it is a vital part of the system. Finding the right expression for how this H field is formed in the magnetic transmission line is helped when we compare it with the classical twin wire line. The math behind the distributed permeance (the inverse of reluctance) shunting the lines is identical to the math for distributed capacitance in the classical case, see Figure 1.

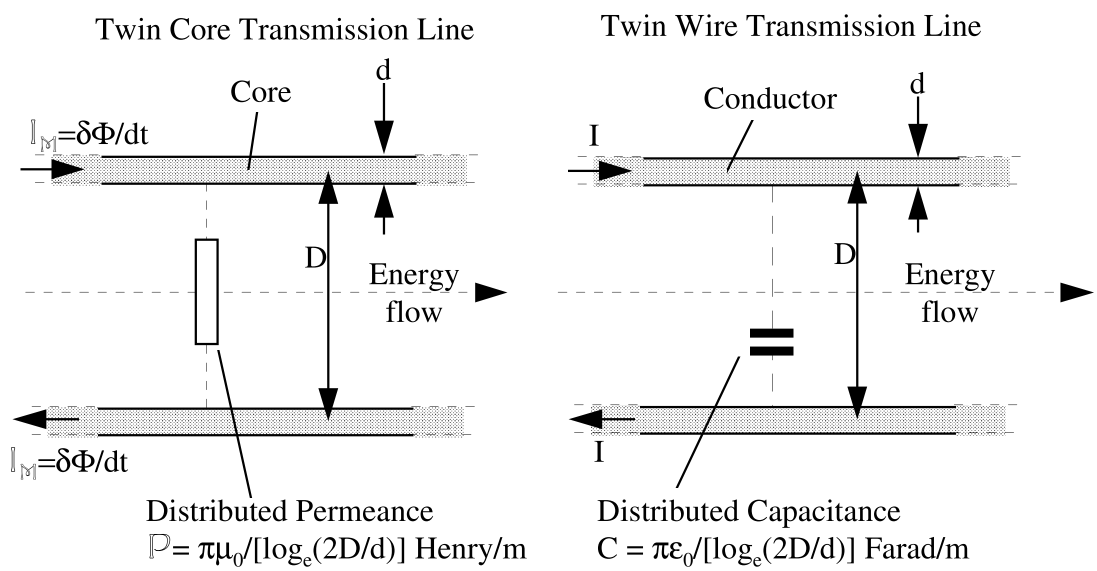

Figure 1. Twin Core Magnetic Line and Twin Wire Transmission Line

In the twin wire line, the distributed capacitance is given by $C=\pi\varepsilon_0/[\cosh^{-1}(D/d)]$ farad/m, where d is the wire diameter and D the separation. For D>>d this becomes $C\approx\pi\varepsilon_0/[\log_e(2D/d)]$. We can therefore immediately deduce that the shunt distributed permeance in the magnetic line is $\mathbb{P}\approx\pi\mu_0/[\log_e(2D/d)]$ henry/m.

Equivalent Circuit

It is well known that magnetic circuits can be represented by electrical equivalents, where flux is modelled as current and mmf is modelled as voltage. Reluctance then plays the part of resistance, leading to "magnetic Ohm's Law". This analogy can be extended so as to perform dynamic analysis of the magnetic circuit in exactly the same manner that is familiar in electrics. To do this the equivalent circuit needs extending to include such things as the effect of a coil connected to a capacitance. This is quite simple, the voltage across the coil (and therefore across the capacitor) is related to flux by $V=N\cdot(d\Phi/dt)$, and the current through the capacitor (and therefore the coil) is related to voltage by $I=C\cdot(dV/dt)$. Therefore the back mmf $N\cdot I$ applied to the magnetic circuit is given by $U=-N^2C\cdot(d^2\Phi/dt^2)$. The capacitively loaded coil looks like a special impedance, something like an inductance (a *magnetic* inductance that is, where we use the letter $\mathbb{L}$ in a different font), but whereas an inductance would offer a back mmf given by the first differential (with respect to time) of the flux, $U=-\mathbb{L}d\Phi/dt$ (just like in electrics $V=-Ldi/dt$), this impedance offers a back mmf related to the second differential of the flux. Let us give the letter $\mathbb{D}$ to represent this impedance. Hence $U=-\mathbb{D}\cdot(d^2\Phi/dt^2)$ where $\mathbb{D}=N^2C$.

We can now construct the electrical equivalent circuit for our delay line.

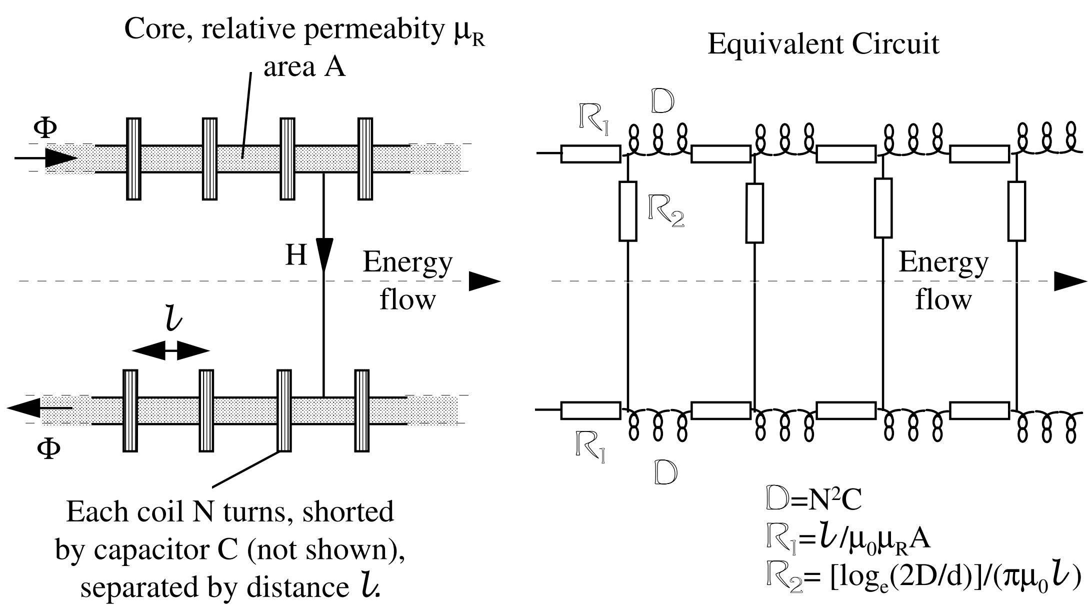

Figure 2. Magnetic Delay Line and Equivalent Circuit

The effect of back mmf's at particular points on the core is to throw out *leakage* H lines, so the equivalent circuit must model these. The series reluctance $\mathbb{R}_1$ is the reluctance of each core segment between the coils, and $\mathbb{R}_2$ accounts for this H *leakage*. This electrical equivalent circuit can be analysed quite easily, but there is an alternative method which gives the answer directly from known examples.

Alternative Equivalent Circuit

We can avoid the second differential for the impedance $\mathbb{D}$ if we adopt a different method of modelling. It has been shown by several authors that it is possible to visualise a *magnetic current* as similar to Maxwell's *displacement current* as applied to dielectrics. Here, instead of modelling flux as current we model the time rate of change of flux instead. Writing $\mathbb{I}_{\mathbb{M}}$ as the equivalent magnetic current, then $\mathbb{I}_{\mathbb{M}}=d\Phi/dt$. When we adopt this model, a reluctance become a capacitance, and our special impedance $\mathbb{D}$ becomes an inductor $\mathbb{L}$.

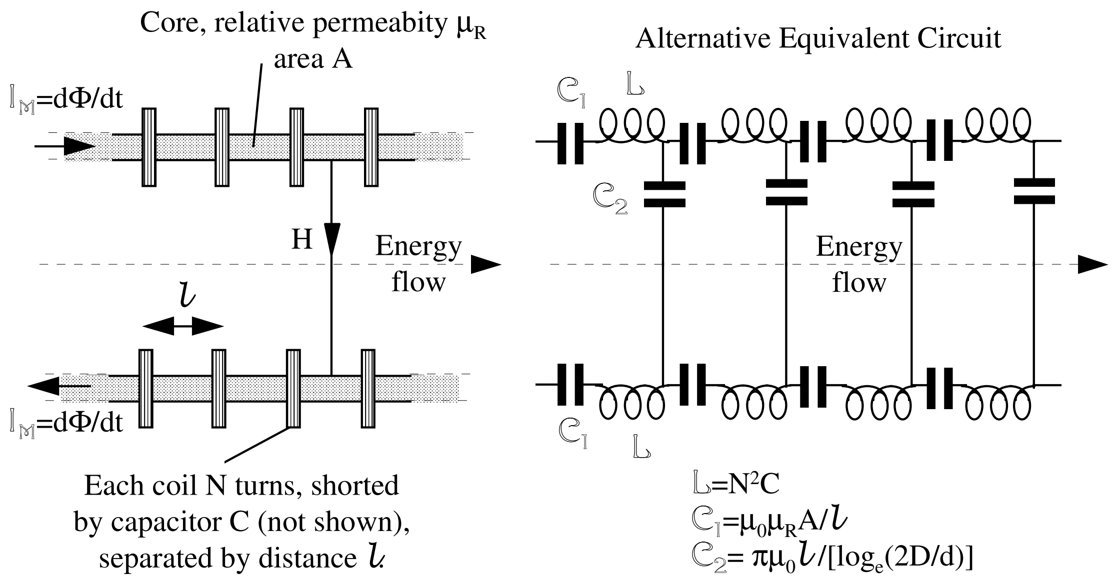

Figure 3. Magnetic Delay Line and Alternative Equivalent Circuit

This equivalent circuit is readily seen as the familiar electrical delay line. This model also helps us realise the importance of the leakage H, represented by the shunt capacitor values $\mathbb{C}_2$. We note that for high permeability cores the series capacitors $\mathbb{C}_1$ are much greater in value than the shunt values $\mathbb{C}_2$, so the $\mathbb{C}_1$ look like short circuits.

We can immediately express the time delay per stage as $\tau \approx\sqrt{(2\mathbb{L}\mathbb{C}_2)}$ and the line impedance as $\mathbb{Z}_0 \approx\sqrt{(2\mathbb{L}/\mathbb{C}_2)}$. Note this *magnetic impedance* $\mathbb{Z}_0$ is in the magnetic domain where the $\mathbb{L}$'s relate to actual capacitors and the $\mathbb{C}$'s relate to actual inductance. When we transform this into the electric domain, e.g. by terminating the line with a shunt core having on it a coil connected to a load resistor R_L, we find that R_L reflects into the equivalent circuit as a magnetic resistor $\mathbb{R}=N^2/R_L$. Thus the correct value of load resistor to provide a matched termination of $\mathbb{R}=\mathbb{Z}_0$, is obtained when $R_L=N^2/[\sqrt{(2\mathbb{L}/\mathbb{C}_2)}]$.

The final figure shows a complete magnetic circuit in the form of a transformer with time delay, and its equivalent circuit. This has the series $\mathbb{C}$'s omitted

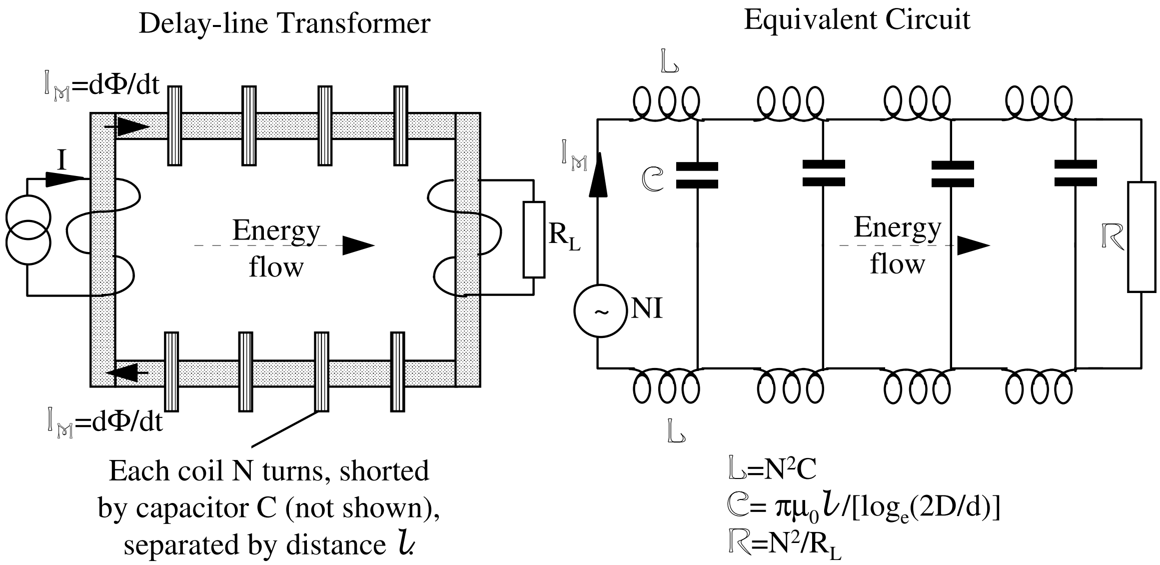

Figure 4. Transformer with Time Delay

Experimental transformers of this type have been constructed, and time delay confirmed. These exhibit classical transmission-line characteristics. When the time delay is one quarter cycle (90° phase shift), a short circuit on the secondary reflects as an open circuit on the primary, and vice versa.

Another situation of interest is when the time delay is one half cycle (180° phase shift). Then for *all* values of load resistance the load is transformed to the primary in the normal manner (another classical transmission-line characteristic). Here we have a transformer where the flux is no longer continuous around the core, the flux in the secondary coil being reversed in direction. You won't find that in the text books!!