More Considerations on PM Generators

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Stan has provided a generator which creates clean sine waves, using a diametrically magnetized ring rotating inside a coil. The use of sine waves helps us in solving the circular problem, the flux in the coil as modified by the Lenz reaction from the load current, is responsible for the voltage which drives the current which creates the Lenz reaction which modifies the coil flux which is responsible for…etc. etc….!! This dilemma is solved by a flux phasor diagram, Figure 1.

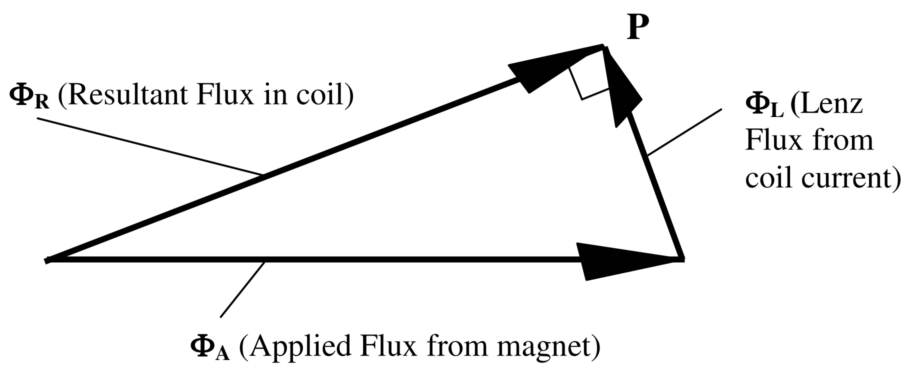


Figure 1

Because the current creating the Lenz reaction is at 90° phase w.r.t. to the resultant flux in the coil, the three flux vectors form a right angle triangle with the applied flux as the hypotenuse. Thus the apex point P must lie on a semi-circle, as shown in Figure 2.

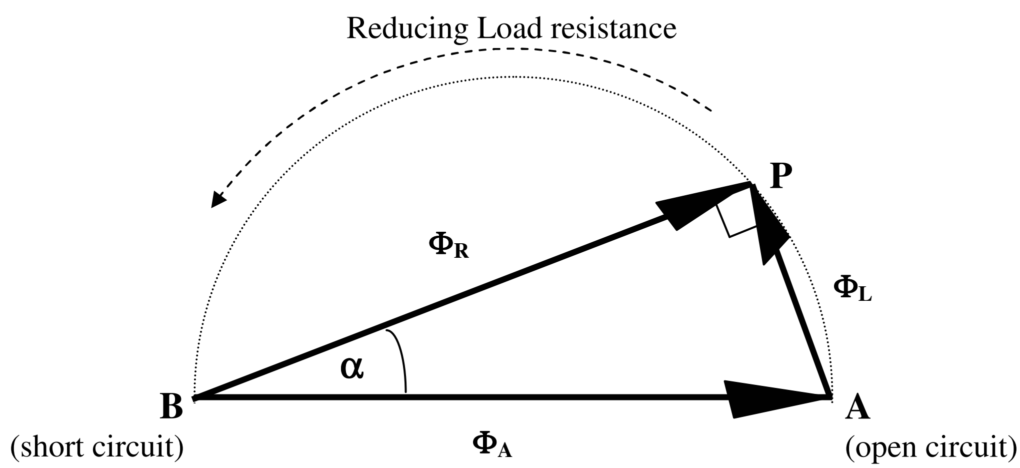


Figure 2

When there is no load present the Lenz flux is zero, P lies to the right of the semi-circle at position A. As load resistance is reduced P moves round the circle, eventually reaching point B when the load is a perfect short circuit. The angle α is a phase shift which increases as the load resistance is reduced.

The problem now is how to calculate the position of P, or the angle α. Clearly α is influenced by an impedance ratio, where we know from experience of other systems $\tan(\alpha)=Z/R$ where R is the load resistance and Z is a system impedance. In this case it is likely that Z is the inductive reactance of the coil, $Z=\omega L$. The phase angle α shifts the voltage waveform w.r.t. the shaft angular position, and this is known as *armature reaction*. This is a phase lag, *so the vectors in figures 1 and 2 are rotating clockwise* (this is opposite to the usual convention for vector rotation, sorry!! To meet the conventional notation the figures should be inverted).

Now we can apply some math to the system. Let the Applied Flux from the magnet be

$$\Phi_A = \Phi_{PK} \sin(\omega t) \qquad (1),$$

where the angle ωt is measured from the magnet position transverse to the coil axis. Then the resultant flux is

$$\Phi_R = \Phi_{PK} \cos(\alpha)\sin(\omega t - \alpha) \qquad (2).$$

We have to differentiate (2) to get induced voltage as

$$V = -\omega \Phi_{PK} N \cos(\alpha)\cos(\omega t - \alpha) \qquad (3)$$

where N is the number of turns. Current I=V/R and this yields flux of value LI/N, so Lenz flux is

$$\Phi_L = \frac{-\omega L \Phi_{PK} \cos(\alpha)\cos(\omega t - \alpha)}{R} \qquad (4)$$

But $\omega L/R=\tan(\alpha)$ so (4) becomes

$$\Phi_L = -\Phi_{PK} \sin(\alpha)\cos(\omega t - \alpha) \qquad (5)$$

which verifies our guess for the impedance ratio, since this fits the vector triangle.

Torque T on the magnet is given by

$$T = -mB\cos(\omega t) \qquad (6)$$

where B is the Lenz field and m is the magnet moment. Taking A as the cross section of the coil then $B=\Phi_L/A$ so using (4) for Φ_L equation (6) becomes

$$T = \frac{m\Phi_{PK} \sin(\alpha)\cos(\omega t)\cos(\omega t - \alpha)}{A} \qquad (7).$$

If the magnet has area A and length $\boldsymbol{l}$ matching the coil, then knowing m is given by M*volume where M is the magnetization, and since $M=B_{REM}/\mu_0$, and $\Phi_{PK}=B_{REM}*A$, we get

$$m = \frac{\Phi_{PK}\boldsymbol{l}}{\mu_0} \qquad (8)$$

Putting this into (7) and multiplying by ω to get shaft power gives

$$P_{SHAFT} = \frac{\omega \Phi_{PK}{}^2 \boldsymbol{l} \sin(\alpha)\cos(\omega t)\cos(\omega t - \alpha)}{\mu_0 A} \qquad (9)$$

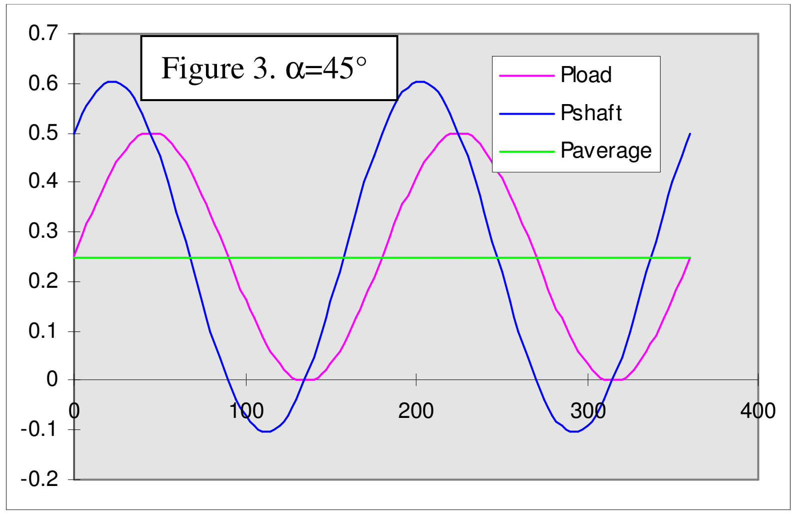
$\boldsymbol{l}/\mu_0 A$ will be recognized as the reluctance of our geometrically matched coil, so from the inductance formula $L=N^2\mu_0 A/\boldsymbol{l}$ we can eliminate the reluctance from (9). Also we can use $\tan(\alpha)=\omega L/R$ to get

$$P_{SHAFT} = \frac{\omega^2 \Phi_{PK}{}^2 N^2 \cos(\alpha)\cos(\omega t)\cos(\omega t - \alpha)}{R} \qquad (10)$$

Now take the power into the load given by V^2/R, V taken from (3).

$$P_{LOAD} = \frac{\omega^2 \Phi_{PK}{}^2 N^2 \cos^2(\alpha)\cos^2(\omega t - \alpha)}{R} \qquad (11)$$

When the phase angle α is very small, (10) and (11) become identical, the shaft power exactly follows the load power. At larger phase angles, although (10) and (11) have the same average value (which will satisfy OU sceptics), the waveforms do not coincide. *Power to the load is not delivered coincidently with power taken from the shaft.* To the classical electrical engineer, this deviation is of little consequence, rotor inertia smooths-out the shaft-power impulses so he is only interested in the average value. Also, from an efficiency point of view, only the average of the load power is of interest. Thus the power impulses are usually ignored. Figure 3 shows P_{LOAD} and P_{SHAFT} for a phase shift of 45°, normalised to $\omega^2\Phi_{PK}^2N^2/R=1$. Figure 4 is for a phase shift of 75°. Note how shaft peak power exceeds load peak. Also note the negative excursions on the shaft power waveform. Over parts of the cycle the rotor torque is negative, the system feeds energy back to the drive source. *What happens if the load is connected only during that part of the cycle where the shaft power is negative?*



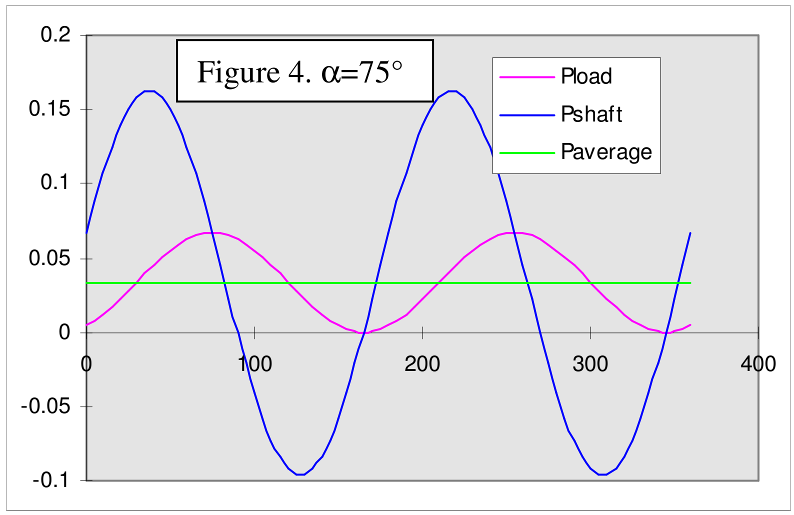
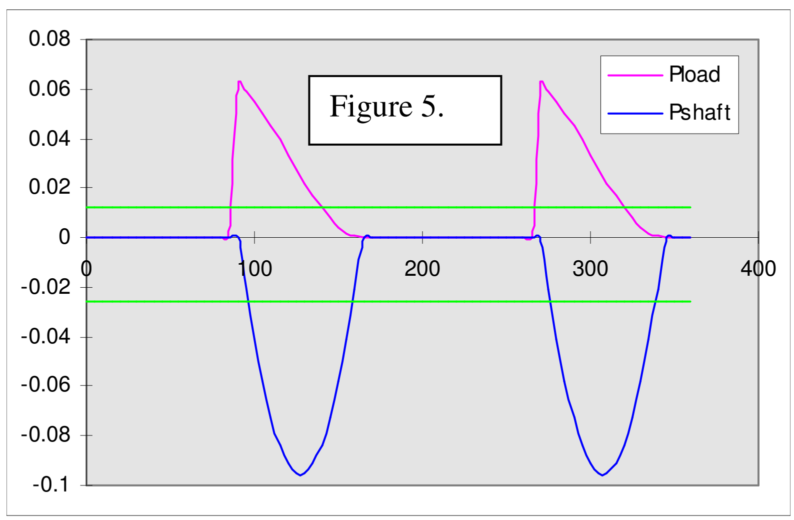


Figure 5 is constructed from Figure 4 (phase shift=75°) by assuming instant waveform jumps, and clearly indicates OU, but it requires a full transient analysis to determine the real waveforms. However before embarking on this journey we need to introduce changes to the magnet due to the external (Lenz) field it sees. The math so far assumes that the magnet's B_{REM} is constant, whereas we know it is not. The first step is to calculate the flux as "seen" by the magnet.



The Lenz flux (5) gets to the magnet but is "chopped" by the rotation, so the magnet sees the flux (5) multiplied by sin(ωt)

$$\Phi_M = -\Phi_{PK}\sin(\alpha)\sin(\omega t)\cos(\omega t-\alpha) \qquad (12)$$

which can be manipulated to

$$\Phi_M = \frac{-\Phi_{PK}\sin(\alpha)[\sin(2\omega t+\alpha)+\sin(\alpha)]}{2} \qquad (13)$$

Note that this has a DC term, the magnet gets a DC bias field which increases with phase angle.

$$\Phi_{DC} = \frac{-\Phi_{PK} \sin^2(\alpha)}{2} \qquad (14)$$

Has this DC field been discovered before? Can it explain OU behaviour?

It may seem strange that a DC field can appear to emanate from a coil, but in reality the coil creates an AC field which can get rectified by the chopping action of the magnet's rotation, so if the chopping action synchronises with the right part of the AC waveform the magnet "sees" this DC flux level. When the phase shift is 45°, the DC term is ¼ of the peak flux from the magnet. At α=75° the DC term is 0.467 of the peak flux.