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Strength versus conductivity

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Abstract

In the development of conductors for high-field coils one has to find a compromise between high strength (to withstand the Lorentz force) and high conductivity (longer pulse time, minimal power, energy and volume required). The design of coils is a non-linear process that is partly based on educated guesses guided by trial and error. Based on the concept of the “current carrying capability” a simple scheme for optimal choice of wire for the relatively simple cases of energy limited coils (or the innermost parts of power limited coils) is developed. On basis of this criterion, the optimal direction of research on new wires or materials is determined.

1. Introduction

The generation of higher magnetic fields is limited by the materials properties, conductivity and tensile strength. The Lorentz force (per volume), which is equal to the product of the current density (j) and the (local) magnetic field, causes a mainly tangential stress, proportional to j^2 (or B^2) in a coil. This stress increases rapidly if one wants to increase the maximum field of a given coil by merely increasing the current density. The same is true for the heating (per volume) that also scales as j^2 .

Unfortunately, conductivity tends to decrease for increasing mechanical strength and a balance between these two properties has to be found. Generally speaking, an optimized magnet (for a given field, inner bore and pulse duration) should consist of wires that are just strong enough so that the mechanical integrity of the magnet is assured while the conductivity is as high as possible.

2. Some simple rules

A more detailed analysis of the adiabatic Joule heating $j^2(t)\rho(T)\Delta t = C_v(T)\Delta T$ shows that not the conductivity itself but the action integral is the most relevant parameter. This approach was used in the design of the Amsterdam 40 T magnet [1]. For the qualification of materials and for the assessment of the combinations of different materials, we introduce the current carrying capability, which is based on the action integral and is defined as

$$\Gamma = \sqrt{\int j^2(t) \partial t} = \sqrt{\int_{T_{\text{begin}}}^{T_{\text{end}}} \frac{C_v(T)}{\rho(T)} \partial T}. \quad (1)$$

The current carrying capability measures the current density a conductor can sustain during a certain period while its temperature rises adiabatically from an initial temperature (T_{begin}) to a maximum allowable temperature (T_{end}).

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3. General mixture rules

3.1. Mechanical properties

The rule of mixtures for mechanical properties are straightforward as the force a composite can handle is the sum of the individual forces on each component:

$$\sigma_{\text{UTS_AB}} = \lambda \sigma_{\text{UTS_A}} + (1 - \lambda) \sigma_{\text{UTS_B}}, \quad (2)$$

where λ is the fraction of material A. It is assumed that both materials will reach their ultimate strength which is the case if one of the materials deforms plastically.

3.2. Current carrying capability

For composite wires (and to a lesser extent for combinations of wire and external or internal reinforcement) the following rules for Γ are applicable: for the isothermal condition, with thermal equilibrium between the materials during the pulse, Γ is given by

$$\Gamma_{\text{AB}}^2 = \lambda \times \Gamma_{\text{A}}^2 + (1 - \lambda) \times \Gamma_{\text{B}}^2. \quad (3)$$

This is based on the assumption that the specific heat is the same for both materials.

In the adiabatic case both materials will have different temperatures. The rules of mixture are derived from the current densities in both materials which are connected electrically in parallel:

$$\Gamma_{\text{AB}} = \lambda \times \Gamma_{\text{A}} + (1 - \lambda) \times \Gamma_{\text{B}}^* \quad (\text{for } \Gamma_{\text{A}} \geq \Gamma_{\text{B}}). \quad (4)$$

The end temperature of materials B is independent of λ for a given value of the end temperature of material A; Γ_{B}^* relates to the final temperature of component B which is lower than T_{end} , and can be approximated by $\Gamma_{\text{B}}^2 / \Gamma_{\text{A}}$.

Γ_{AB} for the adiabatic case becomes very low for small values of λ . An alternative constraint for the adiabatic case is to limit not the temperature of the hottest wire (the best conductor!) but the temperature of the combination of the two materials after relaxation to thermal equilibrium. This condition, called relaxed adiabatic, results in intermediate values of Γ_{AB} that can be approximated by

$$\Gamma_{\text{AB}} = \lambda \times \Gamma_{\text{A}} + (1 - \lambda) \times \Gamma_{\text{B}}. \quad (5)$$

In most cases (especially for short pulses), the adiabatic case is a realistic approximation of the actual Γ , only for material with a relatively high thermal conductivity and for longer pulse-times, or a finely dispersed mixture, a transition towards the relaxed adiabatic case (Eq. (5)) can be expected. Apart from the micro-composites this

situation might be realized in a copper stainless steel conductor with very fine copper filaments.

3.3. General

These general rules of mixtures for Γ and σ allow the comparison of various wire materials and their combinations regarding their suitability for use in a given part of a high-field coil. Assuming that two materials can be combined in any ratio, which is of course not always technically feasible, any intermediate value of Γ and σ_{UTS} can be realized by interpolating linearly or quadratically between both pure materials. A plot of Γ versus σ_{UTS} is therefore a useful way to choose wire materials; in Fig. 1 several pure materials as well as composites are represented. Most of these materials are commercially available, at least in small quantities (25 kg). Only the macro-composites of stainless steel and CuAg or the isothermic version of the stainless steel reinforced Cu are not yet available.

4. Choice of conductor

For choosing the appropriate conductor for high-field coils, we distinguish two cases based on the power supply, which is considered to be either power limited or energy limited.

4.1. Long-pulse (power limited) coils

In the power limited case it is efficient to use several nested coils that are energized consecutively [2]. Two different, extreme situations can be distinguished if the total coil assembly is energized from a power-limited supply:

Case A. The outermost coil: The minimum power condition can be calculated analytically or obtained using a graphical method [2] and results in an optimum volume for a coil with a given inner radius, field and wire-material. Such a coil is normally not stress limited and the best conductor is simply the conductor with the lowest resistivity; in most cases this corresponds to the highest Γ .

Case B. The innermost coil: The issue in the design of an inner coil is to minimize the outer radius of the coil without overheating or overstressing the windings since a smaller outer radius of the inner coil reduces the power in the outer coils enormously.

This case can be solved easily in an analytical way: assume we want to maximize the current density (and thus the additional field ΔB) of a thin (sub) coil of radius r during a time τ and in an external background field B_0 ,

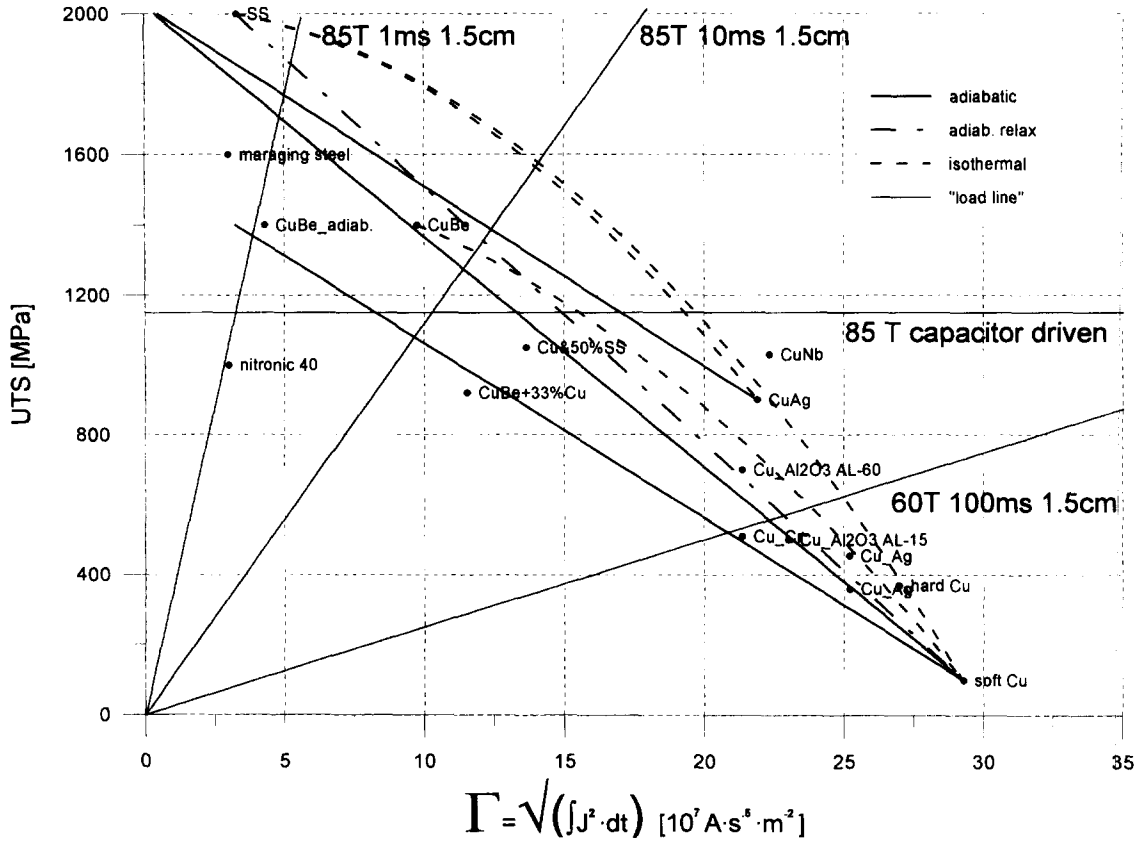


Fig. 1. Properties of materials: conventional materials: copper (Cu), stainless steel (SS); dispersion hardened materials: copper beryllium and Glydco; microcomposites: CuAg [3] and CuNb[4]; macrocomposites: Cu + CuBe, Cu + SS [5] and CuAg + SS. Lines through the origin are “loaded lines” for various high-field sub coils. All other lines correspond to a combination of SS with various conductors, except for the two lines that end at 1400 MPa and represent the combination of CuBe with Cu.

we have

$$\sigma_{\max} \approx B_0 r j, \quad \Gamma^2 = \int j^2 dt \approx 1.3 \tau j^2$$

$$\Rightarrow \sigma_{\max} \approx \frac{B_0 r \Gamma}{\sqrt{1.3 \tau}}. \quad (6)$$

This relation is indicated as “load line” in the figure. In this approximation, the action integral of the pulse is taken to be 1.3 times the action integral during the flat top [2].

Case C. The intermediate coils: In between these two extrema one finds oneself in a case where the increase in power, when the inner coil is made somewhat smaller, has to be balanced against the reduction in power of the outer coils.

In this case, the power of the total coil set has to be minimized, which requires a subtle balancing between an

approach that would lead to minimal volume (case B) and the approach that leads to minimum power of the subcoil (case A). Therefore, these coils need a somewhat less strong and more conducting wire.

4.2. Capacitor driven (energy limited) coils

The magnetic (reactive) energy in a coil is proportional to the volume of the coil and to the magnetic energy density $B^2/2\mu_0$. The energy limitations for capacitor driven magnets therefore impose a constraint on the volume of the coil, which is reflected in the pulse duration.

For low field coils, where the magnetic energy is lower than the heat capacity of the coil, a large fraction of the available energy can be converted into Joule heat during the rising part of the pulse. This gives more freedom for variations of the pulse shape.

For high-field coils, typically above 50–60 T, the magnetic energy stored in the magnet is larger than the heat capacity which imposes stricter limits on the losses due to Joule heat. Part of the reactive energy has to be extracted from the magnet by the end of the pulse and these magnets do not allow a strong crowbar. The magnet is an inductive load for the power supply and the pulse has the shape of a half sine.

Specially for the case of high-field magnets where most of the energy will be converted into magnetic energy, the volume of the coil is constrained by the available energy and the required magnetic energy density, in casu the required field. As a consequence of the geometry of the coil being determined by the available energy, the stress in the coil and the average current density (limiting via the Joule heating the maximum pulse duration) are fixed and cannot be traded one for the other.

The average stress in the coil is of the order $0.4 \times B^2/2\mu_0$ [6, 7] and depends primarily on the geometry of the magnet and to a moderate degree on the type of winding: compared to coils with uniform winding, a reduction of the average stress can be expected in magnets with radially graded internal reinforcement or multi-section coils. The average stress σ imposes a lower boundary condition on the strength of the materials. The material requirement for a energy limited coil is therefore represented by a horizontal line at $\sigma = 0.4 \times B^2/2\mu_0$ in Fig. 1: any material with a strength higher than σ , hence located above the line, will be suitable. A good alternative is a combination of materials such that the average strength is higher than the required level σ . These combinations are represented by the straight thick lines for the rules of mixture in Fig. 1. As mentioned in the first paragraph, the optimum material will be just strong enough to cope with the mechanical load, and have the highest current carrying capability. For magnets made with mixtures of conductor and reinforcement, the optimum combination will be found in Fig. 1 at the intersection of the mechanical load line σ and the dashed line for the rule of mixture behaviour. This intersection gives the current carrying capability I of the mixture and, as the average current density j is determined by the geometry of the magnet, it allows to determine the maximum pulse duration Δt of the magnet from

$$I = \sqrt{j^2 \xi \Delta t}, \quad (7)$$

where ξ is a form factor for the pulse, which is of the order 1/2.

Capacitor driven magnets can also be energized consecutively. This implies that effects of mutual inductance have to be taken into account. Moreover, different parts of the magnet have different rates of heating. As the power from a capacitor bank is generally not a limiting factor, these parts can also be energized with a pulse duration equal to the shortest pulse of the subcoil. This represents a case where the effective pulse duration is the same, but where the material properties are not used to their full potential as one subcoil will not heat up to its limits. This is not an optimal situation and a better performance can be obtained by modifying the stresses and current densities in the subcoils so that no gain can be made by making some pulses of the subcoils shorter, i.e., by making them all equal. The best pulse performance for capacitor driven magnets is thus normally obtained by monolithically energizing the magnet.

5. Conclusion

The definition of generalized rules of mixtures for I and σ_{UTS} permits a systematic choice of optimum wire material for an energy or volume limited (sub)coil.

The dashed parabolic extrapolations in Fig. 1 show that if finely divided copper-stainless steel (and thus more or less isothermal) composite wire could be made for various λ , without compromising the conductivity of the copper, its performance in a magnet coil would be very hard to improve by more exotic types of wire material.

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