CLAUSTROPHOBIC PHYSICS: An introduction to the theory of relativity and Poincaré Symmetry

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14.8.1 Current bearing wires

An ordinary electric current in a metallic wire consists of the movement of negatively charged electrons. The electrons move with a "drift velocity" of order of a few mm/s. Also in the wire are stationary positive charges, residing on the atoms in the metal from which the electrons came.

In the rest frame of the wire, the wire is electrically neutral. The separation between the electrons is not Lorentz contracted because there are only a finite number of electrons in the wire and charge is conserved. Thus the electrons remain spread out over the entire wire.

Let the linear charge density of positive charges be λ_+ and that of the electrons be λ_-. Because the wire is electrically neutral we have

$$\lambda_+ = -\lambda_- \equiv \lambda_0 \tag{14.80}$$

so that the net charge density is

$$\lambda = \lambda_+ + \lambda_- = 0. \tag{14.81}$$

Thus the electric field about the wire is zero and a test charge will experience no electric force.

There will however be a magnetic field surrounding the wire due to the moving electrons. If the drift velocity of the electrons is v then by Eqs. 14.78 and 14.61 the magnetic field is of magnitude

$$B = \left(\frac{k_B k_E}{c}\right) \frac{\lambda_0 v}{2\pi r}. \tag{14.82}$$

Thus a moving test charge q will experience a magnetic force. If the test charge is moving parallel to the wire with the same speed, v, as the electrons then the magnitude of this force is

$$F_m = q\frac{\beta}{k_B}B = q\beta^2 k_E \frac{\lambda_0}{2\pi r}. \tag{14.83}$$

One observes that this is just β^2 times what the electric force on the test charge would be if there were only positive charges in the wire, i.e.

$$F_e = qE = qk_E \frac{\lambda_0}{2\pi r}. \tag{14.84}$$

This is no accident.

Consider the forces on the test charge in its own rest frame (and that of the electrons). In this frame there is no magnetic force. However there is an electric force because in this frame the wire is not electrically neutral! To see this, consider the charge densities of the positive charges and the electrons. We have

$$\lambda'_+ = \frac{Q_+}{l'_+} = \gamma\frac{Q_+}{l_+} = \gamma\lambda_+ = \gamma\lambda_0 \tag{14.85}$$

because of Lorentz contraction of the moving positive charges. However, in this frame the electrons are stationary and their separation is a proper length. Thus

$$\lambda'_- = \frac{Q_-}{l'_-} = \frac{1}{\gamma}\frac{Q_-}{l_-} = \frac{\lambda_-}{\gamma} = -\frac{\lambda_0}{\gamma}. \tag{14.86}$$

This is depicted schematically in fig. 14.7. The net charge density is therefore

$$\lambda' = \lambda'_+ + \lambda'_- = (\gamma - \frac{1}{\gamma})\lambda_0 = \beta^2\gamma\lambda_0. \tag{14.87}$$

Hence, in the rest frame of the test charge there is an electric field

$$E' = k_E\frac{\lambda'}{2\pi r} = \beta^2\gamma k_E \frac{\lambda_0}{2\pi r}. \tag{14.88}$$

The electric force on the test charge is

$$F'_e = qE' = q\beta^2\gamma k_E \frac{\lambda_0}{2\pi r}, \tag{14.89}$$

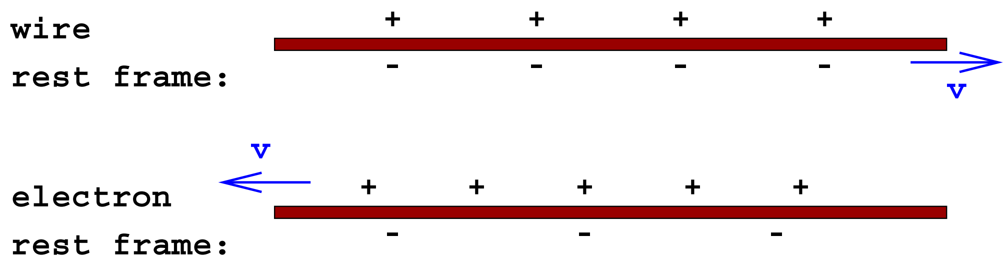

Figure 14.7: A current bearing wire is electrically neutral in its rest frame but appears to carry a net charge in other frames, such as a frame where the conduction electrons are at rest.

pointing radially away from the wire. If we transform this force back to the rest frame of the wire, using the inverses of Eqs. 13.36 or 13.37 with $u'_x = 0$, we get

$$F = \frac{F'_e}{\gamma} = q\beta^2 k_E \frac{\lambda_0}{2\pi r} \tag{14.90}$$

and this is precisely the magnetic force found earlier.

So the magnetic force experienced by the test charge is correctly understood as arising from an electric force in another frame due to a line of net charge density $\gamma\beta^2\lambda_0$, which would correspond to a effective charge density $\beta^2\lambda_0$ in the rest frame of the wire. The relative strength of the magnetic and electric forces is thus of the order of β^2 where β is determined by the drift velocity of the electrons. As remarked above, this is of the order of just a few mm/s and so

$$\beta \approx 10^{-11} \tag{14.91}$$

for typical current bearing wires. The magnetic force they exert is therefore just 10^{-22} times that of an electric force due to just the positive (or negative) charges. That the magnetic force is observed at all (and is appreciable) is due to the enormous amount of charge carried by a number of electrons of the order of Avogadro's number. That this magnetic force is not completely swamped by a still larger electric force is due to the exact cancellation of the positive and negative charges in the wire.