Parallel-Flux Parametric Voltage Regulator and Comparison with Orthogonal-Flux Parametric Voltage Regulator

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Abstract—**An analysis of the parallel-flux parametric voltage regulator shows that it offers all the desirable characteristics of the orthogonal-flux parametric voltage regulator and that it offers several additional advantages. The most striking additional advantage is a much higher energy transfer capability, which implies higher efficiency in material utilization.**

INTRODUCTION

IN A PREVIOUS paper [1] we analyzed an orthogonal-flux parametric voltage regulator. We have also shown [2] that there is a great qualitative similarity between orthogonal-flux interaction and parallel-flux interaction. We should expect, therefore, that a parametric voltage regulator using parallel-flux interaction would have similar properties to one using orthogonal-flux interaction. In this paper we shall investigate the operation and characteristics of such a parallel-flux regulator and compare the characteristics with those of the orthogonal-flux regulator.

ANALYSIS

Consider the schematic diagram shown in Fig. 1. The primary side of the circuit is connected to a voltage source and the secondary side to a load. The winding polarities are shown in the figure. In the analysis we shall use symbols as defined in the figure. Subscripts a and b shall refer to the cores, and subscripts p and s to the primary and secondary sides, respectively. We shall assume zero winding resistance. Capacitor leakage can be considered part of the load resistor.

Let the core material magnetization curve be given by

$$H = a'B + b'B^3. \tag{1}$$

Using the relationships

$$H = \frac{N_w i}{L_m} \quad \text{and} \quad B = \phi/A_c$$

where H, B, N_w, L_m, i, ϕ, and A_c are, respectively, magnetic field intensity, magnetic flux density, number of turns of winding, magnetic path length, current, magnetic flux, and core cross section, we can write

$$N_w i_a = g\phi_a + h\phi_a^3$$

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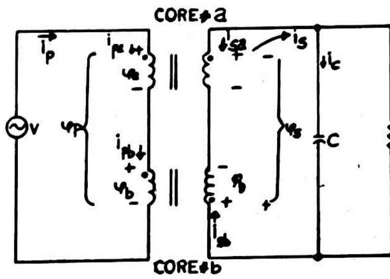

Fig. 1. Schematic diagram of parallel-flux voltage regulator.

and

$$N_w i_b = g\phi_a + h\phi_b^3$$

where

$$g = a'\frac{L_m}{A_c} \quad \text{and} \quad h = b'\frac{L_m}{A_c^3}.$$

In terms of the primary and secondary currents, these equations are

$$N_w i_{pa} + N_{ws} i_{sa} = g\phi_a + h\phi_a^3 \tag{2}$$

and

$$N_{wp} i_{pb} + N_{ws} i_{sb} = g\phi_b + h\phi_b^3. \tag{3}$$

Noting that

$$i_{pa} = i_{pb} = i_p$$

and

$$i_{sa} = -i_{sb} = -i_s$$

one obtains, by subtracting (3) from (2),

$$-2N_{ws} i_s = g(\phi_a - \phi_b) + h(\phi_a^3 - \phi_b^3). \tag{4}$$

Using algebraic identities we find

$$\begin{aligned}\phi_a^3 - \phi_b^3 &= (\phi_a - \phi_b)(\phi_a^2 + \phi_a\phi_b + \phi_b^2)\\ &= (\phi_a - \phi_b)[(\phi_a + \phi_b)^2 - \phi_a\phi_b]\\ &= (\phi_a - \phi_b)(\phi_p^2 - \phi_a\phi_b)\end{aligned}$$

and noting that $\phi_a - \phi_b = -\phi_s$, (4) can be written

$$2N_{ws} i_s = g\phi_s + h\phi_s(\phi_p^2 - \phi_a\phi_b). \tag{5}$$

To express the product $\phi_a\phi_b$ in terms of primary and secondary flux, we note that

$$\phi_p = \phi_a + \phi_b$$

and

$$\phi_s = \phi_b - \phi_a$$

from which

$$\phi_a\phi_b = \tfrac{1}{4}(\phi_p^2 - \phi_s^2).$$

Substituting in (5),

$$N_{ws}i_s + \left(\frac{g}{2} + \tfrac{3}{8}h\phi_p^2\right)\phi_s - \frac{h}{8}\phi_s^3. \qquad (6)$$

To express i_s in terms of ϕ_s we write, from the secondary circuit,

$$\phi_s = \phi_b - \phi_a \qquad i_s = i_c + i_R$$

$$i_R = -\frac{N_{ws}}{R}\dot{\phi}_s \qquad i_c = -N_{ws}C\ddot{\phi}_s.$$

Combining these equations gives

$$i_s = -N_{ws}\left(\frac{\dot{\phi}_s}{R} + C\ddot{\phi}_s\right).$$

Substituting in (6)

$$\ddot{\phi}_s + \frac{1}{RC}\dot{\phi}_s + \left(\frac{g}{2N_{ws}^2C} + \frac{3h}{8N_{ws}^2C}\phi_p^2\right)\phi_s$$

$$+ \frac{h}{8N_{ws}^2C}\phi_s^3 = 0.$$

For sinusoidal input, $\phi_p = \phi_{mp}\cos\omega t$, this equation takes the form

$$\ddot{\phi}_s + Q\dot{\phi}_s + (M + N + N\cos 2\omega t)\phi_s + P\phi_s^3 = 0 \quad (7)$$

where

$$M = \frac{g}{2N_{ws}^2C} \qquad N = \frac{3h}{16N_{ws}^2C}\Phi_{mp}^2$$

$$P = \frac{h}{8N_{ws}^2C} \qquad Q = \frac{1}{RC}.$$

This equation has exactly the same form as [1, eq. (13)] for the orthogonal-flux voltage regulator. This has two important implications. First, it tells us that the two systems have similar characteristics and that they operate on the same physical principles. The energy is transferred by parametric action, the inductance varying at twice the input frequency, while the output frequency is the same as the input frequency. Second, there is no need to engage in an elaborate solution of the equation, but relevant characteristics can be deduced directly from the solution and characteristics of the orthogonal-flux regulator. We merely must replace certain symbols in the expressions for the orthogonal-flux regulator with the corresponding symbols for the parallel-flux regulator. To this end we define the following three additional constants in analogy to corresponding constants of the orthogonal-flux case:

$$K = \frac{N}{M+N} \qquad S = \frac{P}{M+N} \qquad U = \frac{Q}{(M+N)^{1/2}}.$$

The corresponding symbols are as follows.

Orthogonal Flux		Parallel Flux
a	$\leftarrow\ \rightarrow$	g
b	$\leftarrow\ \rightarrow$	h
A	$\leftarrow\ \rightarrow$	M
B	$\leftarrow\ \rightarrow$	N
F	$\leftarrow\ \rightarrow$	K
G	$\leftarrow\ \rightarrow$	S
H	$\leftarrow\ \rightarrow$	U

We can now deduce expressions for the output voltage, automatic turn-off conditions, phase angle between input and output voltages, and required capacitance by referring to the corresponding equations in [1]. By analogy to [1, eq. (19)], the peak value of the steady-state secondary flux is

$$\Phi_{sss}^2 = \frac{2}{3S}(K^2 - 4U^2)^{1/2}.$$

For no-load conditions ($R = \infty$)

$$\Phi_{sss}^2 = \frac{2}{3}\frac{K}{S} = \frac{2}{3}\frac{N}{P} = \Phi_{mp}^2.$$

For the voltage we have, therefore,

$$V_{ms} = \frac{N_{ws}}{N_{wp}}V_{mp}. \qquad (8)$$

In analogy to [1, eq. (18)] we have the condition

$2U \leq K$ to sustain oscillations

from which we obtain

$$R_{\min} = \frac{32N_{ws}^2\omega^3A_c^3N_{wp}^2}{3b'L_mV_{mp}^2}. \qquad (9)$$

A larger load will cause the regulator output to turn off. For a given load R, this equation can be used to determine the input voltage limit below which the out voltage will turn off.

In analogy to [1, eq. (20)], the equation for the phase shift between input and output voltages is

$$\alpha = \tfrac{1}{2}\arcsin\left(-\frac{2U}{K}\right)$$

or

$$\alpha = \tfrac{1}{2}\arcsin\left(-\frac{32N_{ws}^2\omega^3N_{wp}^2}{3RhV_{mp}^2}\right). \qquad (10)$$

For $R = \infty$ this gives a phase shift

$$\alpha = 90°$$

In analogy to [1, eq. (16)], we can evaluate the required capacitor from the relation

$$(M + N)^{1/2} = \omega.$$

This gives

$$C = \frac{1}{2N_{ws}^2\omega^2}(g + \tfrac{3}{8}\Phi_{mp}^2h). \qquad (11)$$

The statement made in [1] concerning the validity of the derived equations is also correct for the present case. We illustrated there that for a silicon–iron magnetic core with a 2-mil air gap the nonlinearity is not quite small enough to allow the derived equations to predict exact numerical values for design. The equations are, however, valid to explain the operation and characteristics of the regulators, to investigate the influences of the various design parameters, and to compare different designs.

COMPARISON OF BOTH TYPES OF REGULATORS

For the comparison we shall introduce the subscript "o" for the orthogonal-flux regulator, and the subscript "pa" for the parallel-flux regulator. As a common base for comparison we require the same iron cross-sectional areas, the same magnetic path lengths, and the same number of turns for the primary windings. This implies that the regulators have the same iron volumes and weights, and that both systems can support the same line voltage with the same degree of saturation under no-load conditions.

As mentioned in [2], the arrangement of Fig. 1 can be replaced with stacked cores or with a single three-legged core. The symbols were, however, defined for the two separate core arrangement so that A_c is the cross section of one core. To be consistent with the basis for comparison we must set

$$A_{co} = 2A_{cpa}. \tag{12}$$

Considering the definitions of g and h in this paper and of a and b in [1], it follows that

$$g = 2a \qquad \text{and} \qquad h = 8b. \tag{13}$$

Voltage: Comparing the corresponding expressions for the voltages, we find, for the same number of secondary turns,

$$V_{ms\,pa} = \sqrt{3}V_{mso}.$$

Thus, for the same design parameters and the same load, the parallel-flux regulator is capable of transferring three times more power.

If it is desired to have the same output voltage for both systems, one must design

$$N_{wso} = \sqrt{3}N_{wspa}.$$

Capacitor: To compare (11) with the capacitor for the orthogonal-flux regulator it is convenient to express the values in terms of the same symbols. Thus we substitute $g = 2a$ and $h = 8b$ so that

$$C_{pa} = \frac{1}{2N_{ws}^2\omega^2}\,(2a + 3\Phi_{mp}^2 b).$$

This shows that $C_{pa} > C_o$. If we wish to obtain the same output voltage for both cases, we must reduce the number of turns by a factor of $\sqrt{3}$, which requires an increase of C_{pa} by a factor of 3.

Minimum Load Resistor (condition for the existence of an output voltage): To compare (9) with [1, eq. 24] for the orthogonal flux, we shall express (9) in terms of orthogonal-flux parameters. We set $A_{cpa} = \frac{1}{2}A_{co}$, and we make the comparison for equal output voltages, which requires $N_{wspa} = N_{wso}/\sqrt{3}$. Thus

$$R_{pa\,min} = \frac{8(N_{wso}N_{wp})^2(A_{co})^2}{19b'L_mV_{mp}^2}.$$

The comparison shows that

$$R_{pa\,min} = 2/19\,R_{o\,min}.$$

This means that the efficiency of material utilization is considerably higher in the parallel-flux case, as the same iron volume can transfer nearly ten times more energy as given by V_{out}^2/R_{min}.

Phase Angle: Expressing (10) in terms of orthogonal-flux regulator parameters under the same conditions as used for the R_{min} comparison, gives

$$\alpha_{ss\,pa} = \tfrac{1}{2}\arcsin\left[-\frac{(2\omega N_{wp}N_{wso})^2}{9RbV_{mp}^2}\right].$$

For both cases we obtain a 90° phase shift between input and output voltages under no load ($R = \infty$). For the same load there is a smaller deviation from the 90° shift in the parallel-flux system.

SUMMARY

We have shown that the parallel-flux regulator offers the same inherent desired properties as the orthogonal-flux regulator—noise rejection and overload and undervoltage protection without auxiliary circuits.

The parallel-flux regulator has a considerably higher energy transfer capability for the same iron volume. This implies a higher efficiency in material utilization. To obtain the same output voltages, the parallel-flux regulator requires less turns for the secondary winding. The phase shift between input and output is less load-dependent in the parallel-flux regulator. For similar conditions, the parallel-flux regulator requires a larger size capacitor. The useful characteristics of both regulators result from the nonlinear ferroresonant output circuits containing variable reactors, rather than from the particular mode of flux interaction.

REFERENCES

[1] Z. H. Meiksin, "Orthogonal-flux parametric voltage regulator," this issue, pp. 424–427.

[2] ——, "Comparison of orthogonal- and parallel-flux variable inductors," this issue, pp. 417–423.

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