# Physical Interpretation of Displacement Current

Mohammed Wazir Adain
Department of Electrical and Electronics Engineering
University College of Engineering
Osmania University, Hyderabad
Telangana, India
mwadain@gmail.com

*Abstract* — **This document depicts a physical view of the current through a capacitor (parallel plate capacitor to be specific). Mathematical proofs and stimulation results have been provided to support the idea. We start by taking a very physical interpretation of the current through the capacitor and then end up proving them. It was actually the other way round that the Displacement Current was discovered. Mathematical equations were experimented to prove is validity. In this paper an idea is presented which provides us the physical essence of current through the capacitor.**

***Keywords-displacemnet current; electric field lines; electric flux lines; dielectric; electric flux density(D); intensity of electric field(E).***

## I. Introduction

The very essence of Science and Engineering is to observe various natural processes and then convert those observations into a Mathematical form usually in the form of laws. Scientists and Engineers perform experiments and the based on their observations they formulate mathematical relations. Discovery of Displacement current is one of the few examples where mathematical relations paved a way to experimental research. James Clerk Maxwell's equations [1] were experimented by Hendrich Rudolf Hertz. The way Displacement Current is usually introduced to the first timers is in the same way. This paper provides a way contrary to this. A physical interpretation of Displacement Current is given which is supported by Mathematical proves and stimulation results.

## II. Perspective of Electron Forced into Capacitor

Let us consider an electron which is forced into the plates of a capacitor plate by some force like an external voltage as in Figure 1. If we view the surroundings from the view of that electron then the electron is pushed into the capacitor plates through the connecting wire. It is repelled by electrons already existing in the plates, but then this electron is the one before the system reaches stability, so this electron gets into the plate. Now once it reaches the plate, it reaches a point where there is no electrical medium for the electron to move further. Thus it reaches point where its journey through the capacitor ceases at least for some time. If we look into the scenario energetically the energy require for the electron to get from the wire to the capacitor plate is very small and the energy required for it to get through the dielectric is very high which cannot be supplied by the voltage source forcing the electron into the plates.

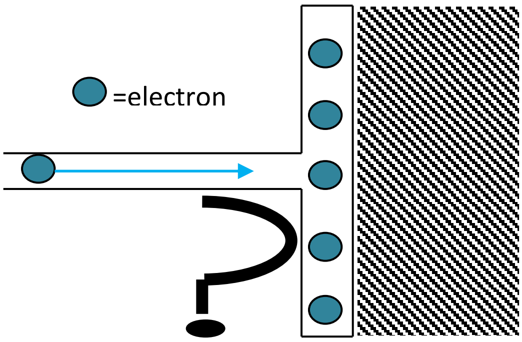


Figure 1. Perspective of electron forced into capacitor

There is no way now that the electron can move, so there is no electron conduction current through the capacitor. Here something very primitive happens. These electrons induce a positive charge onto the other side of the capacitor or they push the electrons on the other side of the capacitor plates away from them and pull the positive charges towards them such that opposite chargers are induced on either side of electron. An electric field is developed in the dielectric medium as in Figure 2. But these charges don't combine as there is an energy barrier between them.

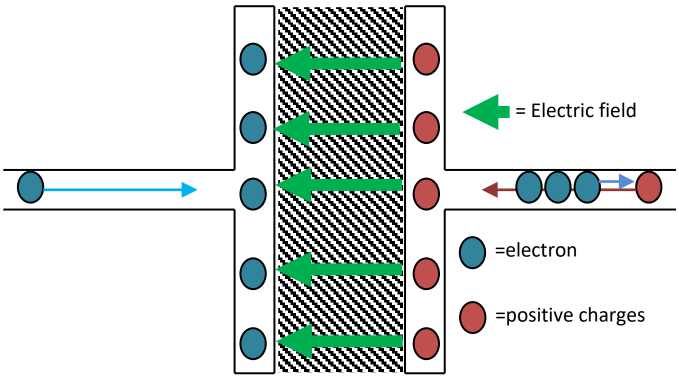


Figure 2. Induction of charges in a capacitor

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## III. Role of Electric Flux Density in Displacement Current

Electric flux density is the electric flux per unit area. Another point of view of looking at D is to imagine that to be electric field lines per unit area. More the field of lines more are the electrons on the other side effected. More field lines imply more flux density. Thus, more electrons are pushed through the capacitor plate (now onwards in the document the capacitor plate refers to the plate on the right side of Figure 1, 2 and 3 unless otherwise mentioned). If we consider an area of cross section ABCD as shown in figure then the current through the capacitor plate is same as the current through ABCD as in Figure 3. In the figure the lines indicate the lines of force on electrons which are opposite to field lines. This is done to illustrate the exact motion of charges and to reduce any ambiguity regarding the motion of positive charges. So once there are electrons pushed through capacitor plates, there are some electrons moving through it in some time, thus there is some current flowing through it. This current sustains for a very small time, indeed an infinitely small time as the electrostatic field through a dielectric stabilizes the charges. The charges attain a position wherefrom no further motion is possible. These static conditions stabilize the system to a static state and the current ceases to flow through ABCD as no charges cannot be accumulated in a conductor. A large value of D can push a large number of electrons through ABCD but it cannot sustain the current flow.

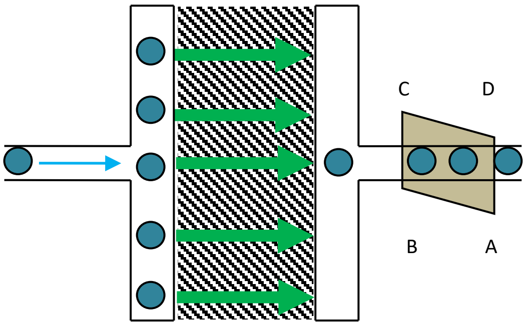


Figure 3. Force on electrons on the capacitor plates and their motion through cross section ABCD

## IV. Analogous Relation with Photoelectric Effect

Intensity of light is the power of light energy per unit area or the light ray flux density. It is the number of light rays per unit area. higher the number of light rays striking the photo metal, higher is the number of photo electrons, thus higher is the photocurrent as shown in Figure 4 [2], similarly higher the value of D more the number of electrons pushed away from the capacitor plate through ABCD. A linear relation exists between photocurrent and intensity of light. The intensity of flux density does not change with time to produce electric current. In is actually the interaction between the photons and the electrons that causes the electrons to eject from the metal surface, but in the case of capacitors the electric field is not strong enough for the electrons to eject out of the metal surface, moreover they are pushed inside through ABCD away from the capacitor plate. This is the part where the our comparison fails, the first part of the comparison holds well and gives a very good picture regarding the relation of D in capacitor current.

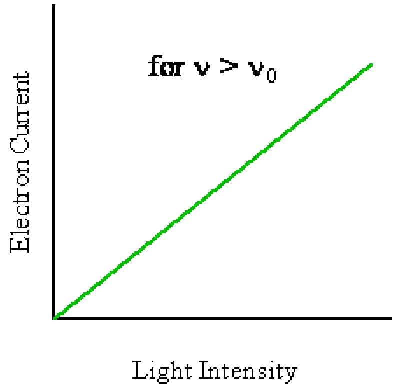


Figure 4. Relation between electron current and Intensity of light

## V. Role of Time Variance of D in Current Through Capacitor

In the last section we had seen that a static D produces no current through the capacitor, thus we try varying D with time. D when varies with time tries to accelerate the electrons but the collisions with the stationary positive ions consumes a lot of their kinetic energy and then the D accelerates the electrons, it collides and comes to rest, this causes it to move with a constant velocity in the conductor known as drift velocity($v_d$) [3]. The effect of a time varying D turns out to be electrons moving with a velocity as of drift. This time varying D causes a field of a push pulls type on the electrons if D changes from positive to negative, but if D has no maxima and minima over a time lapse of some t, the field is either of a push type or a pull type. This D cannot increase infinitely, as at a point the electrons in the plate get depleted and if further, D increases with time the dielectric breaks which is exactly the value of dielectric breakdown voltage. Thus D must vary with time to create a field. If D varies rapidly with time (that is $\frac{\partial D}{\partial t}$ is very high), the push pull action on the electron by the field increases proportionally.

Let 'e' be the charge on electron, 'n' be the number of electrons per unit volume, '$v_d$' be the drift velocity of electrons through the conductor; and the volume of consideration in the further analysis is considered so thin that the electrons in the volume are such that they occupy the same plane or the effect of distance on the conductor is negligible. Then the thickness of the differential volume tends to be the diameter of the electron. The rate of change of D with time drifts the 'n' electrons with a velocity equal to drift velocity. The electrons Follow the electric field.

$$\begin{aligned} &\frac{\partial D}{\partial t} \propto n \\ &\frac{\partial D}{\partial t} \propto v_d \\ &\Rightarrow \frac{\partial D}{\partial t} \propto n v_d \end{aligned} \tag{1}$$

D is basically the force which causes the force on electron to drift, but it moves with a constant velocity as per [3]. So D causes a force on the charge of electron which is 'e' so,

$$\frac{\partial D}{\partial t} \propto e$$

From (1)

$$\frac{\partial D}{\partial t} \propto n e v_d \tag{2}$$

$$\frac{\partial D}{\partial t} = k n e v_d \tag{3}$$

By interpretation the proportionality factor turns out to be 1 as by the analysis so far, we never encountered a factor other than 1. More over if $\frac{\partial D}{\partial t}$ is the force driving the electrons then this force causes n electrons of charge e to drift with a velocity of $v_d$. Thus,

$$\frac{\partial D}{\partial t} = n e v_d \tag{4}$$

Let $\rho_v$ be the volume charge density in the conductor plates, then $\rho_v = ne$; then 4 becomes;

$$\frac{\partial D}{\partial t} = \rho_v v_d \tag{5}$$

By [4],

$$J = \rho_v v_d \tag{6}$$

where J is the current density. Here as the context is that of the current through the capacitor then J id $J_d$, that is displacement current density. Thus,

$$J_d = \rho_v v_d \tag{7}$$

from (5) and (7);

$$J_d = \frac{\partial D}{\partial t} \tag{8}$$

Thus the essence of the document is revealed in equation (8).

As the current through ABCD as in Figure 3 is in series with the capacitor plate, the current through the place is same as the current through ABCD. It is same as the current through the capacitor, which is the displacement current. If we integrate the displacement current density $J_d$ over a surface area S that is the capacitor plate surface area we get displacement current $I_d$;

$$I_d = \int_S J_d \, dS \tag{9}$$

From (8)

$$I_d = \int_S \frac{\partial D}{\partial t} dS \tag{10}$$

If D varies rapidly with time (that is $\frac{\partial D}{\partial t}$ is very high), the push pull action on the electron by the field increases, thus increases the displacement current density and so does the displacement current. This is how current actually displaces through the capacitor plates by electric induction.

But there is a catch here, how is that the electric displacement field or the electric flux density (D) remain conserved over a boundary interface? Let us try applying the boundary conditions for electric field and the electric field density (D). The value of vector D, normal to the boundary interface remains conserved [5]. If we look at the field intensity and flux density by a parallel plate capacitor, both the vectors are perpendicular to the capacitor plates and the dielectric rather they are parallel to the area vector of the capacitor plates. We get across two boundary interfaces actually, the first capacitor plate to dielectric and then the dielectric to the second capacitor plate. In both the cases the D vector is conserved, thus the analysis done so far require no modification. Since the parallel plates are parallel to each other, the flux density D which is perpendicular to one plate is also perpendicular to the other. This analysis also works out even for other type of capacitors as well. If we look into a spherical or a cylindrical capacitor as an example, the field and flux density vectors are perpendicular to the capacitor plates [6][7][8].

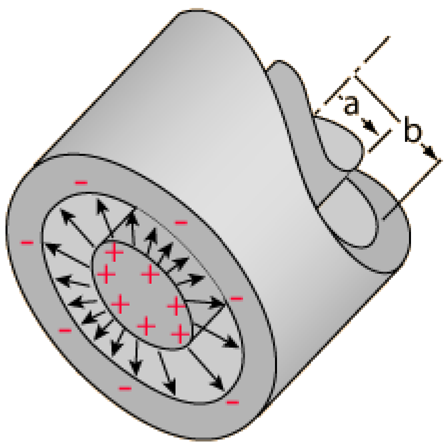


Figure 5. Electric Field and flux density inside a cylindrical capacitor[9]

## VI. STIMULATION RESULTS AND MATHEMATICAL PROOFS

### A. *Stimulation Results*

The circuits shown in the Figure 6 was stimulated in LTSpice and the transient analysis was carried out for the circuit and the output current wave form is shown in the graph which is compared to the input voltage excitation. The input Voltage signal is of 5V amplitude and a frequency of 50Hz.

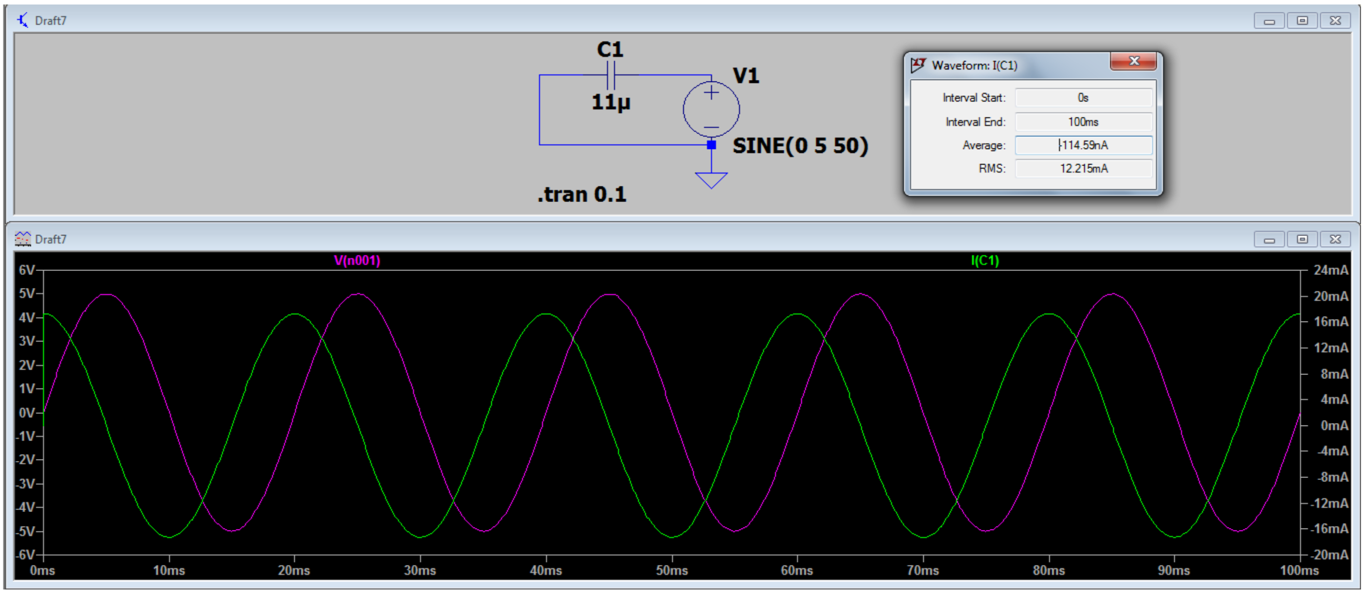


Figure 6. Stimulation Result 1

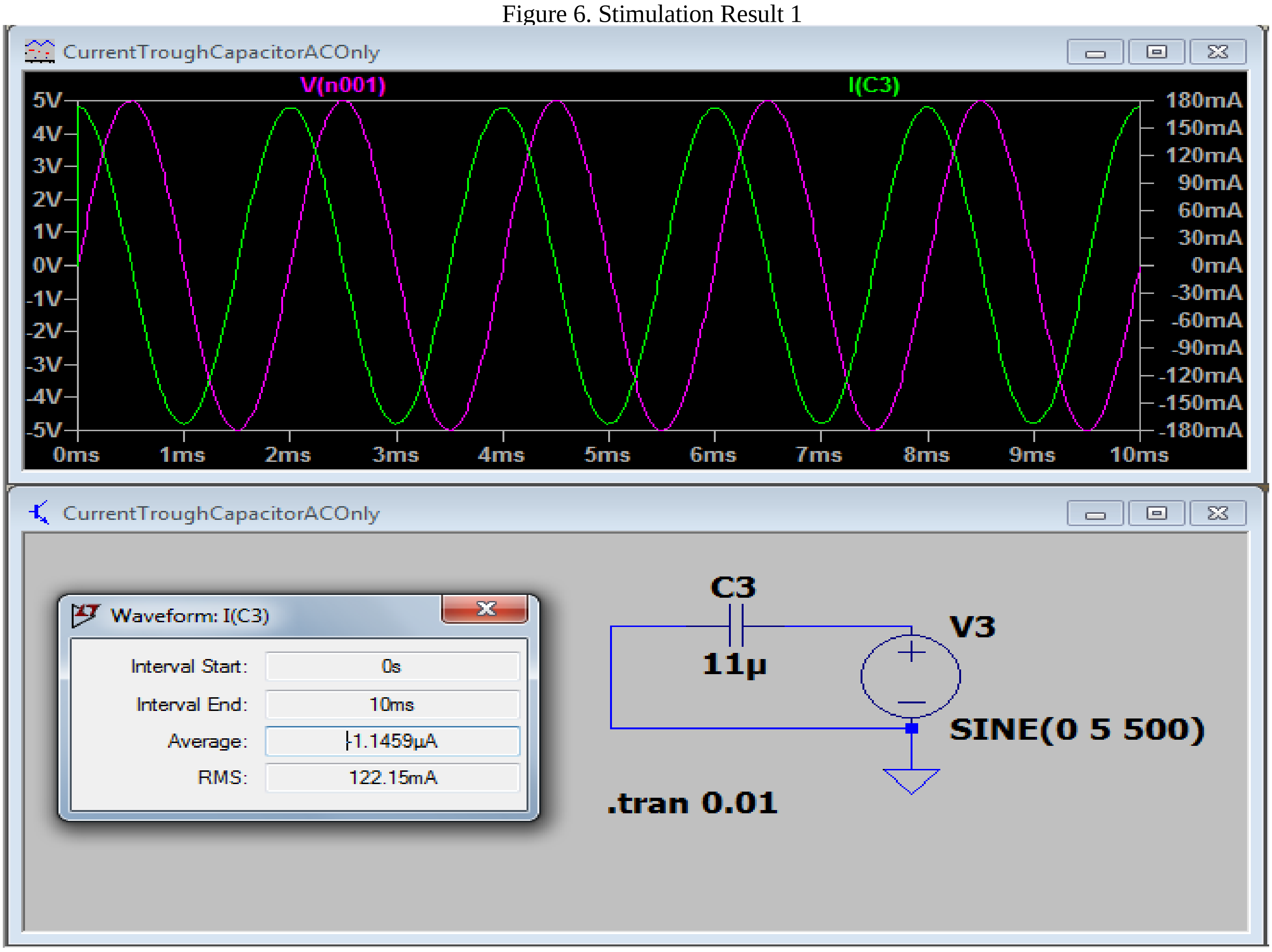


Figure 7. Stimulation Result 2

The rms value of the current through the capacitor in the stimulation in Figure 6 is 12.215mA. A second stimulation was done on the same circuit with the voltage excitation changed such that the frequency is increased tenfold as shown in Figure7. The rms value of the current through the capacitor in this case turns out to be 122.15mA. The rms current value also increases tenfold in the given circuit. Thus, higher the change of D with time in the dielectric of the capacitor, higher is the displacement current. Higher rate of change of D demands a higher value of rate of change of voltage applied across the capacitor which translates to higher frequency of voltage across the capacitor. Thus, the analysis is supported by the stimulation results.

Now if we have a component of the voltage applied across the capacitor which remains constant, then this voltage component cannot excite the capacitor to draw current from it. On the other hand a time varying component of voltage across the capacitor excites it to draw current from this component of the voltage. This is what we can interpret from the analysis done so far. So, let's try to stimulate this idea too. We give an ac voltage with a dc offset to this same circuit as in Figure 6. The stimulation results are depicted in Figure 8.

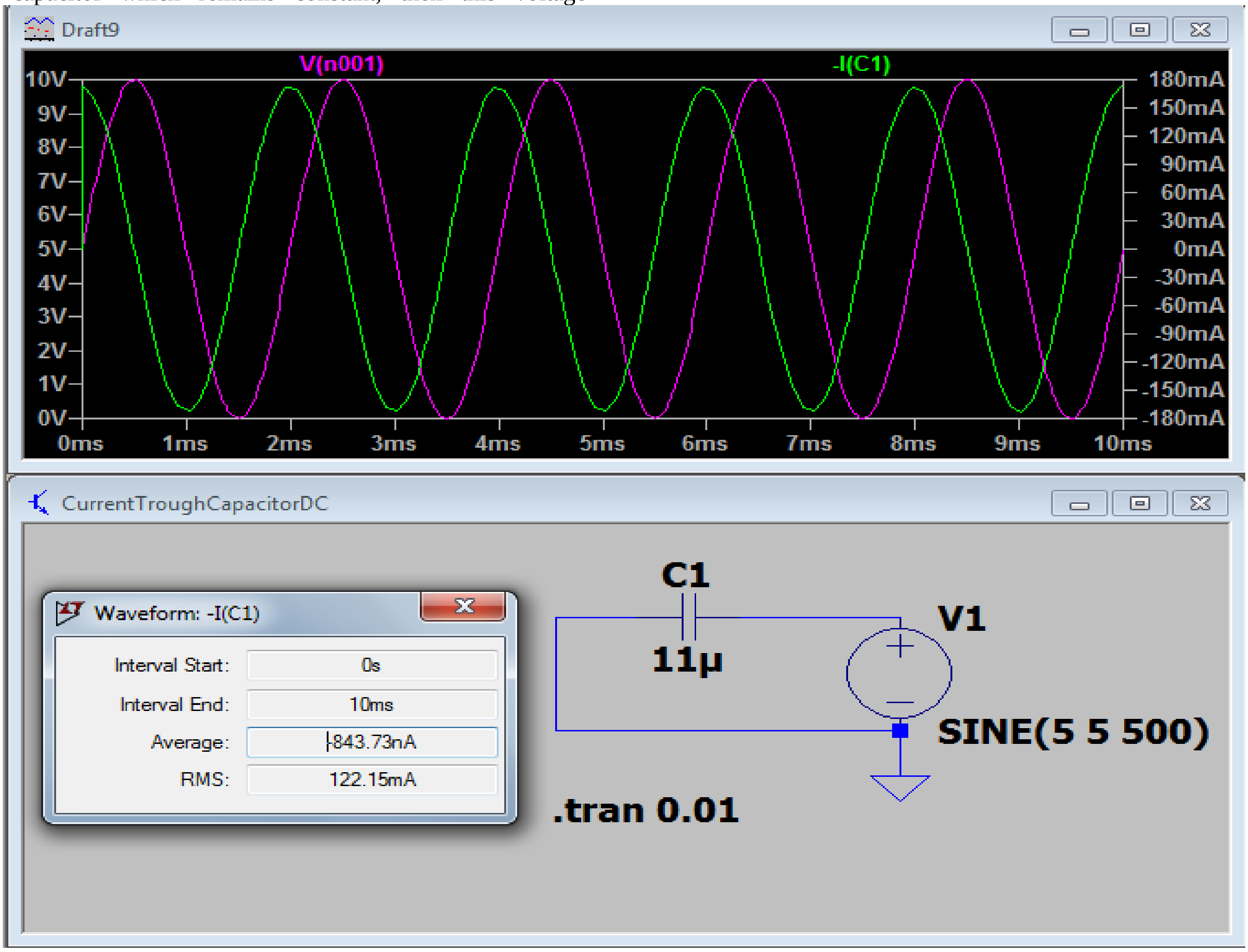


Figure 8. Stimulation Result 3

Even though the voltage excitation has a dc voltage offset, the current through the capacitor has no dc offset (the average value of I= -843.73nA). Thus, the theory presented above is justified by the stimulation results.

### *B. Mathematical Derivation*

A detailed Mathematical proof is given based on some basic laws of vector calculus and the continuity equation [10] as in [11].

## VII. Conclusion

An attempt was made to visualize the displacement current in this document. A physical insight was given into as to how current actually "displaces" through the capacitor was presented. The very primitive idea of electric induction was used to explain the very existence of "displacement current". A relation between displacement current and D and its rate of change with time was presented before the readers. We further see a development of dipole on the plates of capacitor. Stimulation results and mathematical proofs were given to support the idea. Stimulation was done in LTSpice.

## REFERENCES


[1] J. Clerk Maxwell, “A Dynamical Theory of the Electromagnetic Field”, Part III – gerneral Equations of The Electromagnetic Filed, 55-Electrical Displacemenst .

[2] https://chem.libretexts.org/Textbook_Maps/Physical_and_Theoretical_Chemistry_Textbook_Maps/Book%3A_Quantum_States_of_Atoms_and_Molecules_(Zielinksi_et_al.)/02._Foundations_of_Quantum_Mechanics/2.2%3A_Photoelectric_Effect

[3] Matthew N O Sadiku, Elements of Electromagnetism,pp 162-164.

[4] William H. Hayt,Jr. and John A. Buck, Engineering Electromagnetics,Sixth Edition, pp120-121.

[5] Matthew N O Sadiku, Elements of Electromagnetism,pp 182-185.

[6] William H. Hayt,Jr. and John A. Buck, Engineering Electromagnetics,Sixth Edition, pp64-66.

[7] Matthew N O Sadiku, Elements of Electromagnetism,pp 227-228.

[8] NCERT Mathematics Textbook Class 10,pp208-209.

[9] http://hyperphysics.phy-astr.gsu.edu/hbase/electric/capcyl.html

[10] J. Clerk Maxwell, A Treatise on Electricity and Magnetism, 1st ed., vol. 1. Oxford: Clarendon, 1873, pp.342–344.

[11] Matthew N O Sadiku, Elements of Electromagnetism, pp 381-382.