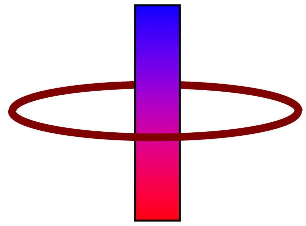

It is wrong to assume the bar magnet's flux that attempts to thread the loop is **zero** when the magnet's center of mass is at the center of the loop and the magnet's magnetization axis is parallel to the loop's axis.

It is wrong to assume **zero flux** in that case just because north and south cancel symmetrically in an incorrectly applied monopole model of the magnet, which violates $\nabla \cdot B = 0$.

In reality, for a real diverging-field-free magnet, the flux through a loop in the exact midplane with coaxial orientation is **non-zero and maximal**.

The mistake comes from assuming that the magnet is two monopole point charges $+q_m$ and $-q_m$ separated by distance 2L, placed symmetrically above and below the plane of the loop. If such classical monopole model is used, the flux comes out to be exactly zero by symmetry, i.e.: every field line that enters the loop from the north pole exits through the same loop from the south pole, or vice versa.

However, a real permanent magnet has no free magnetic monopoles. The divergence of field B is zero everywhere ($\nabla \cdot B = 0$). The correct microscopic picture is a volume (or surface) current density of bound currents, or equivalently, a magnetic dipole moment distribution. For a uniformly magnetized bar magnet, the field outside it is exactly the same as that of a pure dipole ONLY at large distances. Close to the magnet (which is the relevant region here, since the loop is right at the center), the higher-order multipole moments matter, and the monopole term is strictly zero.

The flat disk surface bounded by the loop is NOT a valid surface over which Stokes' theorem can be applied legitimately when using the free-space vector potential A, because that surface is pierced by magnetized material where the simple relation $\nabla \times \mathbf{A} = \mathbf{B}$ no longer holds in the same way.

Let's unpack this carefully.

What Stokes' theorem actually says

Stokes' theorem is $\Phi = \int_S B \cdot dA = \oint_{\partial S} A \cdot dl$

This is **always true**, but the **A** on the right-hand side must be a vector potential that is valid (single-valued and differentiable) over the **entire surface S**.

In free space (or non-magnetized regions), we can always find such an A, and in the Coulomb gauge it satisfies $\nabla \times A = B$ everywhere in the region of interest.

Why the flat disk fails in this specific case

When the bar magnet straddles the plane $z = 0$ and its body **physically intersects** the flat disk bounded by your loop:

- Inside the magnetized material, $B = \mu_0 (H + M)$
- The free-space vector potential A_free that we normally use outside is related only to H by $\nabla \times A\_free = B$ outside, but inside the material $\nabla \times A\_free = \mu_0 H \neq B$.

More fatally for the theorem application:

The magnetization M is odd under $z \to -z$, so the equivalent bound volume current $J\_b = \nabla \times M = 0$ (uniform M), but there is a surface bound current on the lateral surface. The key point is that the **vector potential produced by the magnet has a discontinuity or a branch cut** when you try to cross the magnetized region with your surface.

In the symmetric coaxial case, the vector potential A has only an A_φ component, and A_φ(r, z) is an **odd function of z**. Therefore, on the plane $z = 0$ **outside the magnet**, $A\_\varphi = 0$, so $\oint A \cdot dl = 0$ if you stay outside the magnet. But the flat disk you want to integrate over **goes right through the magnet**, and inside the magnet A_φ is **not zero** — in fact, inside a long uniformly magnetized cylinder, A_φ grows linearly with radius, exactly like inside a solenoid.

So the free-space A that is zero on the midplane outside the magnet **does not continue smoothly** into the magnet; there is a jump or a branch when crossing the magnetized region.

Therefore, you **cannot** use the flat disk as the integration surface with the ordinary free-space vector potential. The mathematical object A is not well-behaved (not single-valued or not differentiable) across the entire disk.

How to compute the flux correctly

You have two valid options:

1. **Choose a different surface** that does not pierce the magnet For example, deform the surface into a hemisphere above the loop plus a hemisphere below. On those surfaces B is not odd in z in the same way, and when you add the contributions, the end-face effects no longer cancel perfectly — you recover the full non-zero flux.
2. **Explicitly use the solenoid equivalence** The bar magnet ≈ solenoid with surface current $K\_\varphi = M$. The amperian loop around the cylinder encloses this current, so by Ampère's law (or Biot–Savart), B_z inside the effective solenoid is uniform and equal to $\mu_0 M$ (for a long magnet). Your conductive

loop encircles exactly this same current (if it's larger than the magnet's radius), so the flux is $B_z \times$ (area of loop minus area of magnet cross-section) + small corrections from the ends → definitely not zero.

Bottom line

The reason the line integral $\oint A \cdot dl$ around the loop appears to give zero while the actual flux is large is that **you are not allowed to use the flat disk as the Stokes surface** when the magnet pierces it. The vector potential you would need to use across that disk is not the ordinary free-space one; it has a discontinuity across the magnetized region.

This is exactly analogous to the famous "solenoid paradox" or the Aharonov–Bohm situation with a confined field: the vector potential outside can give a vanishing line integral while the flux through the enclosed area is μ_0 times the total enclosed (bound) current — which is non-zero.

So in your symmetric configuration, the flux is very real, large, and maximum — the apparent zero from the line integral is an artifact of choosing an invalid integration surface that cuts through the source.