Quote from: Mazuto on Yesterday at 11:34:58:

> In the Lorentz force equation $F = q \cdot (E + v \times B)$, what does the "`v`" refer to ?
> I already know that it is a velocity. I guess it is a `relative velocity` because you told me that `absolute velocities` do not exist (i.e.: all motions are relative) but I do not know between what entities is that relative velocity measured.

It's the velocity of the charged particle relative to the reference frame in which the electric and magnetic fields E and B are measured.

So if you measure E and B in the laboratory frame, then v is the particle's velocity in the laboratory frame. If you measure E and B in some other inertial frame, then v is the particle's velocity in that same frame.

The key insight is that **E and B and v must all be measured in the same reference frame** for the Lorentz force equation to be valid in that frame.

The electric and magnetic fields themselves are frame-dependent. If you boost to a different inertial frame, E and B transform into different fields E' and B' (mixing into each other in the process), and the particle's velocity becomes v'. The beautiful thing is that when you calculate $F' = q(E' + v' \times B')$ in the new frame, you get the correctly transformed force.

There's no such thing as "absolute velocity", within the context of the Lorentz force equation, we simply need to be consistent: pick a reference frame, measure everything in that frame, and the equation works. The "relative to what?" question is answered by: "relative to whatever frame you're using to describe the electromagnetic fields."

Quote from: Mazuto on Yesterday at 13:11:28:

> Does that mean that it does not matter whether the charged particle is moving with respect to the magnetic field or the magnetic field is moving with respect to the charged particle and in both cases the Lorentz force is the same ?

The **total force** on the particle is the same in both frames of reference.

This gets at something really interesting about how electromagnetic fields transform between reference frames.

Let me illustrate with an example:

Scenario 1: Lab frame

- A magnetic field B exists (say, from a stationary magnet)
- A charged particle moves with velocity v through this field
- The particle experiences a magnetic force: $F = qv \times B$

Scenario 2: Particle's rest frame
Now, let's switch to the frame where the particle is at rest:

- The particle has $v' = 0$ in this frame
- So the magnetic force $qv' \times B' = 0$ (no magnetic force on a stationary charge!)
- But the particle still experiences a force (we know it accelerates in the lab frame)

The resolution: When you transform to the particle's rest frame, the magnetic field B doesn't just stay as B. Part of it transforms into an **electric field** E'. This electric field now exerts the force on the (now stationary) particle.

So the key points are:

1. The **total force** on the particle is the same in both frames (after proper relativistic transformation)
2. But the **split between electric and magnetic contributions** is different in different frames
3. What appears as a purely magnetic force in one frame appears as partly (or wholly) electric in another frame

This is one of the beautiful insights of special relativity: electric and magnetic fields are really two aspects of a single electromagnetic field, and they mix into each other when you change reference frames. The Lorentz force equation $F = q(E + v \times B)$ automatically accounts for this, as long as you use the correct E, B, and v for your chosen frame.

Quote from: Mazuto on Yesterday at 16:43:19:

> I do not understand why you wrote:
>
> "Scenario 2: Particle's rest frame Now let's switch to the frame where the particle is at rest: The particle has $v' = 0$ in this frame"
>
> Alas, the velocity between the particle and the B field is not zero because by assuming the charged particle as stationary we are forced to attribute their relative motion to the B field. In other words: the charged particle becomes stationary at the cost of the magnetic field B becoming moving.
> Their relative velocity has the same magnitude as in Scenario 1.

You're correct that the **relative velocity between the particle and the source of the B field** (like a magnet) has the same magnitude in both frames. In Scenario 2, if the particle is at rest, then yes, the magnet creating the B field is now moving.

However, here's the crucial point: **magnetic fields don't "carry" velocity in the sense you might expect.** The magnetic field B at a point in space is just a field value at that point. When we transform to a different reference frame, we're asking "what are the field values E' and B' in this new frame?"

The transformation depends on the relative velocity between frames, but once we know E' and B' in the particle's rest frame, we use those values with the particle's velocity **in that frame**, which is $v' = 0$.

What happens is:

- The moving magnet (in the particle's rest frame) creates both an electric field E' and a magnetic field B' in that frame
- Since $v' = 0$, the magnetic part contributes nothing: $qv' \times B' = 0$
- But $E' \neq 0$ now (it appeared due to the transformation), and this provides the force: $F' = qE'$

The key insight: **the fields themselves (E and B) are what transform between frames**, not by "moving" but by mathematically mixing into each other based on the relative velocity between frames. The magnetic field doesn't move—rather, the pattern of field values throughout space is different when described in different frames.

Quote from: Mazuto on Yesterday at 18:37:27:

Does the particle experience the same force in both of these scenarios ?

The answer is **yes, but with an important caveat about what "same" means.**

The particle experiences the **same physical force** in the sense that:

- The particle's trajectory through spacetime is the same regardless of which frame you use to describe it
- If the particle accelerates in one frame, it accelerates in all frames (though the numerical value of the acceleration will be different due to time dilation and relativistic effects)

However, the **numerical value of the force** does transform between frames according to relativistic transformation rules. For everyday velocities (much less than the speed of light), the force magnitude is approximately the same, but at relativistic speeds, you need to account for how forces transform.

More importantly for your original question: Even at low speeds, what I wrote earlier is the key:

- **Lab frame:** Force is $F = qv \times B$ (purely magnetic)
- **Particle frame:** Force is $F' = qE'$ (purely electric, since $v' = 0$)

These give the same physical effect (the same acceleration of the particle, accounting for frame differences), but the force comes from **different field components** in each frame.

So to directly answer your question: Yes, the particle experiences the same physical force in both scenarios—it's the same physical situation described from two different perspectives. The breakdown into electric versus magnetic contributions changes, but the net effect on the particle's motion is consistent across different frames.

Quote from: Mazuto on Yesterday at 19:07:14:

Considering that the particle experiences the same physical force in both of these scenarios (neglecting relativistic effects at speeds close to c) what is the shape of the electric field in Scenario 2 ?
Assume that the magnet (the source of magnetic field) is moving with respect to the initially stationary charged particle at speed << c. Also, assume a horseshoe magnet or C magnet with the magnetic flux lines in the air gap that are almost parallel mutually and uniform and the magnet is moving perpendicularly to these lines.

Setup for Scenario 2 (particle's rest frame):

- The horseshoe/C-magnet is moving with velocity **v** relative to the (now stationary) charged particle
- In the magnet's rest frame (Scenario 1), there was a uniform magnetic field **B** in the air gap, perpendicular to the magnet's motion in Scenario 2.

The induced electric field E' in Scenario 2:

When the magnet moves, the transformation from the lab frame to the particle frame creates an electric field given approximately by:

$$\mathbf{E'} \approx \mathbf{v} \times \mathbf{B}$$

For your configuration:

- If **B** points perpendicularly to the magnet's velocity **v** (say, B points "across" the gap and v is the direction the magnet moves)
- Then **E' = v × B** points in the third perpendicular direction
- The electric field is **uniform** in the gap region (since B was uniform)
- The magnitude is $|E'| = vB$

Physical picture: The electric field in the gap moves along with the magnet. As the magnet approaches, passes by, and recedes from the stationary charged particle, the particle experiences this moving electric field pattern. When the particle is in the gap region, it experiences a uniform electric field E' that produces exactly the same force as the magnetic force $qv \times B$ did in Scenario 1.

This is essentially a moving magnet inducing an electric field—the same as the principle behind Faraday's law of electromagnetic induction!

Quote from: Mazuto on Yesterday at 21:16:52:

> When the horseshoe/C magnet moves perpendicularly to the magnetic flux lines in its air gap, then is the electric field vB unipolar or bipolar (unidirectional or bidirectional) from the point of view of the charged particle ?

The electric field **E' = v × B** is **unipolar (unidirectional)** from the point of view of the stationary charged particle.

Here's why:

In the gap region:

- The magnetic field **B** is uniform and points in one direction (say, from north pole to south pole across the gap)
- The magnet's velocity **v** is uniform (perpendicular to B)
- Therefore **E' = v × B** points in a single, uniform direction throughout the gap

The electric field doesn't reverse direction—it points consistently in one direction as determined by the cross product $v \times B$.

As the magnet moves past the particle:

- When the gap approaches the particle: E' points in direction $(v \times B)$
- When the gap is at the particle: E' still points in direction $(v \times B)$
- When the gap moves away: E' still points in direction $(v \times B)$ (but the particle is no longer in the strong field region)

The field doesn't flip polarity as the magnet passes—it maintains the same direction. The particle experiences this uniform electric field for the duration that it's within the gap region.

This is **unipolar/unidirectional**.

Note: Outside the gap, the field configuration is more complex, but within the uniform B-field region of the gap, E' remains unidirectional.

Quote from: Mazuto on Yesterday at 22:39:04:

> How do you reconcile that with the Faraday's law of induction that states that when the magnetic flux increases then the induced voltage is in one direction and when the magnetic flux decreases then the induced voltage is in the opposite direction ?

There is a crucial distinction between these two different physical situations:

1. Faraday's Law for a conducting loop: When a magnet moves past a stationary conducting loop:

- As the magnet approaches: flux through the loop increases → induced EMF in one direction
- As the magnet recedes: flux through the loop decreases → induced EMF in the opposite direction
- This is **bipolar** - the EMF and current direction reverses

2. A single charged particle (our scenario): A single charged particle at a fixed point in space experiences:

- The electric field $\mathbf{E'} = \mathbf{v} \times \mathbf{B}$ at that specific location
- This field is **unipolar** - it points in one consistent direction determined by $v \times B$

The reconciliation:

The key is that Faraday's law gives you the **EMF around a closed loop** ($\oint E \cdot dl$), not the electric field at a single point.

When the magnet moves past a conducting loop:

- **Before the gap arrives:** The particle experiences weak/zero field
- **As gap approaches:** The particle enters a region where E' points in direction $(v \times B)$
- **As gap recedes:** The particle exits that region

For a **loop** of wire:

- Different parts of the loop are at different positions relative to the moving magnet
- As the magnet moves, the **front edge** of the gap creates E' in one direction
- The **back edge** creates field transitions in the opposite sense
- The line integral around the loop (the EMF) reverses sign as the magnet passes

So: A single charge sees a unipolar field at its location, but a conducting loop integrates over space and sees a bipolar EMF as the field pattern moves past it. The bipolar nature comes from the **spatial gradient** of the moving field pattern, not from the field reversing direction at a point.

Quote from: Mazuto on Today at 01:08:47:

In the Scenario 2 with the C magnet moving uniformly and perpendicularly to the magnetic flux lines in its air gap (assume parallel and uniform magnetic flux lines in that gap) and the charged particle being first approached, straddled and finally departed by that gap, could you show me the mathematical formalism that proves that:

1) Single charge sees a unipolar field at its location.

2) A conducting loop integrates over space and sees a bipolar EMF as the C magnet's gap moves past it and that the bipolar nature of the EMF comes from the spatial gradient of the moving magnetic field pattern.

Here is the mathematical formalism for both cases:

Setup

Coordinate system:

- Magnet moves in the **+x direction** with velocity **v**
- Magnetic field in the gap points in the **+z direction**: $\mathbf{B} = B_0 \hat{\mathbf{z}}$
- Gap extends from $x = vt - w/2$ to $x = vt + w/2$ (width w, centered at $x = vt$)
- Gap has height h in the y-direction

Magnetic field (in particle's rest frame):

$B(x,t) = B_0 \hat{z}$ for $vt - w/2 < x < vt + w/2$

$B(x,t) = 0$ otherwise

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1) Single Charged Particle at Fixed Position

Electric field from frame transformation:

$E' = v \times B$

At the particle's location (say $x = 0$):

- **Before gap arrives** ($t < -w/2v$): $B = 0$, so $\mathbf{E' = 0}$
- **While in gap** ($-w/2v < t < w/2v$): $B = B_0 \hat{z}$, so $\mathbf{E' = v \times (B_0 \hat{z}) = vB_0 \hat{y}}$
- **After gap departs** ($t > w/2v$): $B = 0$, so $\mathbf{E' = 0}$

Force on charge q:

$F = q E' = q vB_0 \hat{y}$ (while in gap)

$F = 0$ (outside gap)

Key result: The electric field is **unipolar** - it always points in the **+ŷ direction** when present. It doesn't reverse; it simply turns on and off.

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2) Conducting Loop - Bipolar EMF

Consider a rectangular loop in the xy-plane:

- Left edge at $x = -L/2$
- Right edge at $x = +L/2$
- Height h in y-direction

Method 1: Faraday's Law (Flux Method)

Magnetic flux through loop:

$$\Phi(t) = \iint B \cdot dA$$

Case (a): Gap approaching (gap right edge at $x_{gap} = vt < -L/2$):

$$\Phi = 0$$

Case (b): Gap entering ($-L/2 < vt < L/2$): Overlap length: $\Delta x = vt + w/2 - (-L/2) = vt + w/2 + L/2$

$$\Phi(t) = B_0 \cdot h \cdot \min(\Delta x, w, L)$$

$$d\Phi/dt = B_0\ h\ v \quad \text{(positive - flux increasing)}$$

$$EMF = -d\Phi/dt = -B_0\ h\ v \quad \text{(negative, by Lenz's law)}$$

Case (c): Gap fully inside (gap fits entirely within loop, if $w < L$):

$$\Phi = \text{constant} = B_0\ h\ w$$

$$d\Phi/dt = 0$$

$$EMF = 0$$

Case (d): Gap exiting (front edge past $+L/2$, back edge still inside):

$$d\Phi/dt = -B_0\ h\ v \quad \text{(negative - flux decreasing)}$$

$$EMF = -d\Phi/dt = +B_0\ h\ v \quad \text{(positive, opposite sign!)}$$

Result: EMF is **bipolar** - negative when flux increases, positive when flux decreases.

Method 2: Motional EMF (Direct E-field Integration)

The EMF around the loop is:

$$EMF = \oint E' \cdot dl$$

Since $\mathbf{E' = vB_0 \hat{y}}$ (only where $B \neq 0$), and the loop has segments parallel and perpendicular to $\hat{y}$:

- **Top/bottom segments** (parallel to x): $E' \cdot dl = 0$ (perpendicular)
- **Left/right segments** (parallel to y): $E' \cdot dl = \pm vB_0\ dy$

When gap is entering the loop:

- Right edge of loop ($x = +L/2$) is outside gap: $E' = 0$
- Left edge of loop ($x = -L/2$) is inside gap: $E' = vB_0\ \hat{y}$

EMF = ∫(bottom, left side) $E' \cdot dl$ + ∫(other segments) 0

= $-vB_0\ h$ (going down on left side, against E')

When gap is exiting the loop:

- Left edge ($x = -L/2$) is outside gap: $E' = 0$
- Right edge ($x = +L/2$) is inside gap: $E' = vB_0\ \hat{y}$

EMF = ∫(top, right side) $E' \cdot dl$

= $+vB_0\ h$ (going up on right side, with E')

Key insight: The **spatial gradient** of the B-field (the edges of the gap) is what matters. The EMF comes from the fact that:

- **Leading edge** of gap: one side of loop has $E' \neq 0$, other side has $E' = 0$ → net EMF in one direction
- **Trailing edge** of gap: opposite situation → net EMF in opposite direction

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Summarizing

1. **Single charge:** Sees $E' = vB_0\ \hat{y}$ (unipolar) at its location when the field is present
2. **Conducting loop:** Integrates $E' \cdot dl$ around a closed path that spans regions with and without the field. The **bipolar EMF** arises because:
   - The spatial discontinuity ($\partial B/\partial x \neq 0$ at gap edges) means different parts of the loop experience different fields
   - This creates a non-zero line integral $\oint E' \cdot dl$
   - The sign reverses depending on whether the leading or trailing edge is crossing the loop

The mathematics confirms: unipolar field at a point, bipolar EMF around a loop, arising from spatial gradients of the moving field pattern.