Analysis of Time-Harmonic Eddy Currents and Spin Precession

Simulation Report, by Que

April 22, 2026

1 Introduction

This report details the relationship between time-harmonic magnetic fields, induced eddy currents in non-ferrous conductors, and the resulting phase-dependent spin dynamics.

2 Advanced Eddy Current and Spin Dynamics

The interaction between an AC master magnet and a non-ferrous metal depends on the spatial distribution of the induced field. In a copper sphere, the induced circulatory eddy currents change direction as the field from the AC master electromagnet changes direction.

According to Lenz's Law, this reversal is essentially a mechanism of opposition. As the master field oscillates at frequency ω, the eddy currents must flip their polarity to create a counter-magnetic field. Because the eddy currents flip at the same frequency as the AC source, there is no relative "flip" between them in the steady state, except for the slight phase lag ϕ.

This phase lag is critical when considering the electron spin state. The permitted state for electron spin is at an angle of 54.7° (the "magic angle") to the magnetic field. The spinning electron acts like a small bar magnet, and when placed at an angle to an applied magnetic field, it is subject to a torque trying to align it with the field.

To maintain the angle at a constant 54.7°, this torque is balanced by a gyroscopic one of equal but opposite magnitude. Therefore, quantum rules force the electron spin-axis to precess around the field direction. When the Helmholtz coils are driven at a frequency equal to the AC master magnet, we observe a "slowed down" Larmor frequency. Within the material, for every skin depth δ you move into the sphere, the phase of this precession shifts by an additional 1 radian: $\phi(z) = \frac{z}{\delta}$

At a depth of 2δ, the phase lag is exactly 2 radians, causing the spin orientation to rotate significantly relative to the surface orientation.

3 Time-Harmonic Current Density and Power Absorption

The current density $J(z)$ at a depth z within the conductor is calculated as a function of the surface current density J_0 and the skin depth δ: $J(z) = J_0 e^{-z/\delta} e^{j(\omega t - z/\delta)}$

In this model:

- **Surface Current Density (J_0):** Defined by the source excitation of the master magnet.
- **Exponential Decay:** The current density decreases by $1/e$ for every skin depth δ of penetration.
- **Linear Phase Shift:** The phase of the current lags linearly with depth, shifting by 1 radian for every δ.

The total power P_{total} absorbed per unit surface area of the copper sphere is the integral of the local Joule heating density from the surface to infinity: $P_{\text{total}} = \int_0^\infty \frac{J_0^2}{\sigma} e^{-2z/\delta} dz = \frac{J_0^2 \delta}{2\sigma}$

This relationship holds for semi-infinite conductors where the radius is much larger than the skin depth ($R \gg \delta$). In the rotating frame (slowed-down Larmor frequency), this total power represents the steady-state energy required to maintain precessing spins against resistive damping.

4 Electromagnetic Fundamentals

The penetration of an AC magnetic field into a conductor is governed by the skin depth, defined as: $\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$

Where:

- δ is the skin depth (m)
- $\omega = 2\pi f$ is the angular frequency (rad/s)
- μ is the magnetic permeability
- σ is the electrical conductivity (S/m)

5 Eddy Current Dynamics and Phase Shift

At a depth z within the conductor, the induced current density $J(z)$ is subject to both exponential attenuation and a linear phase lag: $J(z) = J_0 e^{-z/\delta} e^{j(\omega t - z/\delta)}$

The phase shift ϕ, measured in radians, is proportional to the depth: $\phi = -\frac{z}{\delta}$

This relationship implies that for every skin depth δ of penetration, the phase shifts by exactly 1 radian (approximately 57.3°).

6 Energy Dissipation

The total power P_{total} absorbed per unit surface area of the material is calculated by integrating the local Joule heating losses: $P_{\text{total}} = \frac{J_0^2 \delta}{2\sigma}$

7 Material Comparison Table

The following table compares the electromagnetic and quantum parameters for different non-ferrous materials at an operating frequency of $f = 1000$ Hz.

Material	**Conductivity (σ)**	**Skin Depth (δ)**	**Phase Lag at 1mm**
Copper	5.8×10^7 S/m	2.09 mm	0.48 rad
Aluminum	3.5×10^7 S/m	2.69 mm	0.37 rad
Brass	1.5×10^7 S/m	4.11 mm	0.24 rad

Table 1: Comparison of penetration and phase lag for common conductors.

8 Quantum Context: Slowed Larmor Precession

In a rotating frame synchronized with the AC master magnet, the Larmor frequency is effectively slowed to the Rabi frequency. Because of the depth-dependent phase lag ϕ, the precession axis for electron spins rotates as a function of z, creating a helical orientation profile through the first two skin depths of the copper sphere.

9 Interaction in the Slowed Rotating Frame

In the "slowed down" rotating frame, the relationship between phase lag, skin depth, and precession is primarily found in the interaction between bulk electromagnetic effects and microscopic quantum dynamics.

9.1 Phase Lag and the Precession Axis

The phase lag ϕ directly affects the orientation of the effective field in the rotating frame. In this system:

- **Phase Lag (ϕ):** Determines the timing of the B_1 field in the rotating frame and shifts the azimuthal angle of the precession axis.
- **Skin Depth (δ):** Defines the observation volume for spin precession. Only the spins within the skin depth layer are significantly influenced by the AC master magnet.
- **Precession Angle:** While the "magic angle" (54.7°) is a quantum constraint, the orientation of the field itself rotates as you move deeper into the material.

9.2 The Phase-Depth Relationship

For every skin depth (δ) of penetration, the phase of the AC field shifts by exactly 1 radian. This creates a specific spatial orientation for the spins:

1. At the **surface**, the electron precesses around an effective field with zero additional phase lag.
2. At **one skin depth deep**, the precession is around an effective field that is rotated by 1 radian (57.3°).
3. At **two skin depths deep**, the rotation reaches 2 radians (114.6°).

This depth-dependent phase shift causes the spins throughout the sphere to precess in a **helical pattern** rather than as a uniform ensemble, effectively "twisting" the Larmor precession axis throughout the volume.

The simulation of the copper sphere isn't reacting as one solid block. It's reacting as a series of thin "sheets," each precessing at a different phase.