PLANETARY GEAR SYSTEM ANALYSIS

240T Ring Gear / 200T Sun Gear | Cross Slide Drive

Geometry • Kinematics • Force Transfer

---

1. System Overview

This document describes a planetary-style gear system driven by a cross slide mechanism. The analysis covers geometry, kinematics, and force transfer under ideal conditions. All elements are treated as ideal: no friction, no compliance, and the only force at the gear mesh is the normal force acting on the tooth face.

1.1 Scope of Analysis

- Geometry: pitch radii, center distance, mesh type
- Kinematics: motion relationships, gear ratios, rotation per cycle
- Force transfer: mechanical advantage, structural loads, torque paths
- No other considerations are made at this time

---

2. System Configuration

2.1 Components

The system consists of the following elements:

- Pivot: fixed at coordinate origin (0, 0)
- Arm: 10 mm rigid beam, free to rotate about the pivot
- Pin: fixed to the far end of the arm, cannot rotate relative to the arm
- Ring Gear: 240 tooth, Module 1, mounted on the pin and free to rotate about it
- Torque Device: mounted between the pin and ring gear, can apply or react torque between them
- Sun Gear: 200 tooth, Module 1, mounted on the cross slide — can orbit but cannot rotate

- Cross Slide: provides a 10 mm radius orbital motion centred on (0, 0) — this is the system input

2.2 Initial Positions

Element	Position	Notes
Pivot	(0, 0)	Fixed ground point
Cross slide / Sun gear centre	(0, +10)	Input driver
Arm pin / Ring gear centre	(0, −10)	Diametrically opposite to sun
Centre distance	20 mm	Sun to ring gear centres

---

3. Geometry

3.1 Gear Parameters

Parameter	Sun Gear	Ring Gear
Tooth count	200T	240T
Module	1	1
Pitch radius	100 mm	120 mm
Role	Non-rotating orbiter	Rotating output

3.2 Mesh Type

The center distance between the sun gear and ring gear centers is 20 mm. For an external gear mesh, the required center distance would be:

$$C\_external = R\_sun + R\_ring = 100 + 120 = 220 \text{ mm} \quad ✗$$

This does not match. For an internal (annular) gear mesh:

$$C\_internal = R\_ring - R\_sun = 120 - 100 = 20 \text{ mm} \quad ✓$$

The geometry is therefore confirmed as an internal mesh. The 240T ring gear is an annular (internal) gear, and the 200T sun gear runs inside it. This is consistent with the planetary-style assembly shown in the design.

---

4. Kinematics

4.1 Arm and Cross Slide Relationship

As the cross slide orbits at radius 10 mm, the arm must keep the ring gear centre exactly 20 mm from the sun gear centre at all times. Since both centres orbit at 10 mm from the origin, the only configuration that maintains the 20 mm separation throughout the full cycle is for the arm to point diametrically opposite to the cross slide at all times.

If the cross slide orbit angle is φ, then the arm angle tracks at $\varphi + 180°$. Both rotate at the same rate. This is consistent with the stated initial condition: cross slide at (0, +10) and pin at (0, −10).

4.2 Ring Gear Rotation

The sun gear is rotationally locked to ground through the cross slide mechanism — it cannot rotate. As the arm carries the ring gear around the non-rotating sun gear, the ring gear rolls on the sun gear's teeth.

Applying the standard epicyclic rolling constraint for an internal gear pair with a fixed sun:

$$\omega_{ring/pin} = (T_{sun} / T_{ring}) \times \omega_{arm}$$

$$\omega_{ring/pin} = (200 / 240) \times \omega_{arm} = (5/6) \times \omega_{arm}$$

4.3 Rotation Per Cycle

For one complete orbit of the cross slide (360°):

$$\text{Ring gear rotation relative to pin} = (200/240) \times 360° = 300°$$

The ring gear does not complete a full revolution relative to the pin in a single orbit cycle. The system repeats every 6 cycles, at which point the ring gear has completed exactly 5 full rotations relative to the pin and all elements return to their original relative positions.

4.4 Kinematic Ratio Summary

Input	Output	Ratio
1 full cross slide orbit (360°)	300° ring gear rotation about pin	6 : 5
6 full cross slide orbits	5 full ring gear rotations	6 : 5
Any ω_{orbit}	$(5/6) \times \omega_{orbit}$ at ring/pin	6 : 5

5. Force Transfer

5.1 Assumptions

- All elements are ideal
- No friction at any interface
- At the gear mesh, there is no compressive force between the gears
- All force at the mesh is the normal force acting on the gear tooth face
- Input is applied solely by the cross slide

5.2 Mechanical Advantage

The cross slide drives the sun gear centre in its orbit at a radius of 10 mm from the pin. The tooth normal force acts at the ring gear pitch radius of 120 mm from the pin. Both are moment arms about the same point — the pin.

MA = R_ring / R_orbit = 120 mm / 10 mm = 12 : 1

For every unit of tangential force applied by the cross slide at its 10 mm orbit radius, the ring gear receives 12 times that force as torque at the pin. This is a fixed geometric property of the system and does not vary with arm angle or cycle position.

5.3 The Torque Device

The torque device sits between the pin and the ring gear. The pin is fixed to the arm. Therefore the device sits directly at the pin axis — at 0 mm moment arm from the pin centre.

The device sees the full ring gear torque. It can act as:

- An output — extracting work produced by the cross slide driving through the mesh
- An input — adding torque to assist or drive the ring gear
- A brake — resisting ring gear rotation as a load

The torque balance at the pin is:

F_crossslide × 10 mm + T_device = F_tooth × 120 mm

5.4 The Arm

The arm is a rigid 10 mm beam between the pivot at (0,0) and the pin. It carries two distinct loads:

- Radial load: the mesh reaction force acts along the arm's length as tension or compression between pivot and pin
- Tangential load: the counter-torque from the torque device passes through the pin into the arm as a bending moment, which is reacted at the pivot as a tangential force opposing the direction of arm rotation

The pivot is therefore not passive. It is the torque ground for the entire system, reacting the counter-torque from the device as a tangential force at the pivot bearing. The pivot bearing sees both a radial load (from mesh forces) and a tangential load (from the device counter-torque).

Because the full ring gear torque is reacted over only 10 mm of arm length, the arm carries structural loads 12 times greater than the tooth normal force. Despite being the smallest structural element, the arm is the most highly loaded.

5.5 Force Path Summary

Stage	Element	Load Type
1	Cross slide	Tangential orbital force at 10 mm radius
2	Sun gear / tooth mesh	Normal force at tooth face
3	Ring gear	Torque about pin axis (120 mm × F_tooth)
4	Torque device	Full ring gear torque extracted or applied at pin
5	Pin → Arm	Counter-torque as structural moment in arm
6	Pivot	Tangential reaction force — system torque ground

---

6. Input / Output Relationship

Because the 12:1 mechanical advantage is a fixed geometric property, the relationship between device output torque and required cross slide input is constant and linear:

```
T_input = T_device / 12
F_crossslide = T_device / 120 mm
```

This holds for any output the device produces, at any arm angle, during any cycle.

6.1 Example Values

Device Output Torque	Required Cross Slide Input Torque	Cross Slide Force (at 10 mm radius)
12 N·mm	1 N·mm	0.1 N
120 N·mm	10 N·mm	1 N
1,200 N·mm	100 N·mm	10 N
T N·mm	T / 12 N·mm	T / 120 N

---

7. Consistency with Newtonian and Kinematic Requirements

The system as described is fully consistent with Newtonian mechanics and kinematic constraints:

- Geometry: the internal mesh at 20 mm centre distance satisfies both the tooth counts and the module exactly
- Kinematics: all constraints are honoured throughout the full orbit; the centre distance is maintained continuously; the 6:5 ratio follows directly from the tooth counts
- Force transfer: all forces balance at every joint; no forces are created or destroyed; the pivot grounds the counter-torque tangentially; the arm carries the mesh load radially
- Newton's Third Law is satisfied at every interface: mesh, pin, arm, and pivot

The system is mechanically valid and self-consistent under the stated ideal conditions.

End of Analysis